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A Mixed Integer Linear Programming (MILP) model for Advanced Earthwork Allocation Planning

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Abstract- *This paper presents a MILP model which identifies the optimal cut-fill pairs and their sequence with minimum total earthwork cost. The proposed model is of value to earthwork managers because it identifies the most favorable EAP by accounting for the rock-earth type of each and every rock-earth, the series of rock-earths occupying each and every cut and fill pits, and the moving directions (i.e., the order of cut-fill rock-earth pairs), expeditiously. A test case confirms the usability and validity of the model.*

I. Introduction

Earthwork is engineering processes to change a current ground surface into a desired surface by excavating rock-earth from cut pits and moving it to fill pits. Earthwork allocation planning (EAP) identifies the optimal cut-fill pairs and their sequence to minimize the total earthmoving cost by assigning cuts to fills economically [1,3]. Various EAP methods, which are based on either linear programing or evolutionary algorithms, have been introduced into the earthwork community to identify favorable cut-fill pairs and their sequence [4,5,6]. However, existing studies for EAP did not handle several issues discussed as follows: First, the series of rock-earth of cut pit should be excavated in top-down sequence; the series of rock-earth of fill pit should be embanked in bottom-up sequence. Second, the cut-fill pit pairs and their sequencing should be constrained by taking into account top-down sequence of cut rock-earths and bottom-up sequence fill rock-earths. Third, rock-earth type (or quality) needs to be considered to bank each fill rock-earth of fill pits. For example, a fill pit may have a subgrade, which supports the asphalt paving layer, and a road-bed. Only good quality soil (i.e., less than 100 mm particle-size) can be used to construct the subgrade. The series of cut or fill rock-earths and their soil types could be obtained from geological columnar sections of all pits. Fourth, once excavator positions into a cut pit, it is needed to dig out cut rock-earths as many as possible in order to minimize the excavator's travel distance between cut pits. Therefore, constraints about excavator movement are required to be added in the model. Fifth, when a non-conforming cut rock-earth is transported into a fill rock-earth, secondary blasting, which used to reduce the dimensions of oversized rock-earth, or disposing rock-earth should be required. It also requires additional corrective action cost or disposal cost. A justification should be confirmed by comparing if the disposal cost is smaller than the corrective action cost of the non-confirming cut rock-earth. This paper proposes a MILP model as a viable solution that considers the research gaps identified. A new MILP model which handles these issues with the least cost is presented in this paper.

II. MILP Model for Optimal EAP

2.1 Defining Earthwork Job-Site

An earthwork site is divided into pits. A pit is classified into either a cut ($i \in C$), a fill ($j \in F$), or a balanced pit according to the rock-earth volume required to achieve the planned ground level. If the amount of rock-earth needs to be moved out of the pit (i.e., cut volume: $CV(i)$), then the pit is cut pit (i). If the amount of rock-earth needs to be moved into the pit (i.e., fill volume: $FV(j)$), then the pit is fill pit (j). A cut pit (i) and a fill pit (j) are respectively sliced into a series of cut earth-block and a series of fill earth-block using user-defined earth unit size (the earth-block size). The data structure of the series of cut earth-blocks generated by slicing the i^{th} cut pit by the size is a first-in-first-out (FIFO) queue because these cut earth-blocks are excavated from their corresponding cut pit in top-down order. The data structure of the series of fill earth-blocks generated by slicing the j^{th} fill pit is a last-in-first-out (LIFO) queue because these fill earth-blocks are backfilled into their fill pit in bottom-up order. Also, a set of borrow pits (b) and disposal pits (w) are considered for the earthwork. A borrow pit (b) is an off-site source to import the scarce fill rock-earths, a disposal pit (w) is an off-site location to export the excess cut rock-earths. Each borrow pit has a maximum borrow capacity ($BC(b)$). Each disposal pit also has a maximum waste capacity ($WC(w)$).

2.2 Defining the Earthmoving Input Sets, Parameters and Variables

$X\ (i,j,t)$ is a variable that represents a cut earth-block moved from a cut pit i to a fill pit j at t^{th} earthmoving iteration. $X\ (i,w,t)$ is a variable that represents cut earth-block moved from a cut pit i to a waste pit w at t^{th} earthmoving iteration. $X\ (b,j,t)$ is a variable that represents a cut earth-block moved from a borrow pit b to a fill pit j at t^{th} earthmoving iteration. $C\ (i,j)$ is a value that represents unit earth-block moving cost from a cut pit i to a fill pit j. $c\ (i,w)$ is a value that represents unit earth-block moving cost from a cut pit i to a waste pit w. $c\ (b,j)$ is a value that represents unit earth-block moving cost from a borrow pit b to a fill pit j. This study computes the unit earth-block moving costs ($c\ (i,j)$, $c\ (b,j)$, $c\ (i,w)$) by taking into account these productivity loss factors using Gwak et al.'s [2] approach. $Ax\ (j,t)$ is used to identify if corrective action is occur in j^{th} fill pit at t^{th} earthmoving iteration. $Ac\ (j,t)$ is a value that represents corrective action cost in j^{th} fill pit at t^{th} earthmoving iteration. $Y\ (i,k,t)$ and $p\ (b,n,t)$ indicates whether the k^{th} cut rock-block of the i^{th} cut pit and the n^{th} rock-block of the b^{th} borrow pit moves at the t^{th} earthmoving iteration, respectively. $Z\ (j,k,t)$ and $q\ (w,k,t)$ denote whether the k^{th} fill rock-earth of the j^{th} fill pit and the k^{th} fill rock-earth of the w^{th} disposal pit are banked at the t^{th} earthmoving iteration, respectively.

2.3 Formulating the Objective Function and the Constraints

The MILP model with the objective function of minimizing the total earthmoving cost of rock-earth among cut pits, fill pits, disposal pits, and borrow pits and minimizing corrective action cost of nonconforming rock-earths transported into the fill pit is presented as follows:

$$\text{Minimize Z} = \sum_t\sum_i\sum_j x(i,j,t)\times c(i,j) + \sum_t\sum_i\sum_w x(i,w,t)\times c(i,w) + \sum_t\sum_b\sum_j x(b,j,t)\times c(b,j) + \sum_t\sum_j ax(j,t)\times ac(j,t)$$

Subject to:

$$\sum_t\sum_j x(i,j,t) + \sum_t\sum_w x(i,w,t) = CV(i), \forall i \quad (1)$$
$$\sum_t\sum_j x(i,j,t) + \sum_t\sum_b x(b,j,t) = FV(j), \forall j \quad (2)$$
$$\sum_t\sum_j x(b,j,t) \le BC(b), \forall b \quad (3)$$
$$\sum_t\sum_i x(i,w,t) \le WC(w), \forall w \quad (4)$$
$$\sum_t\sum_j x(i,j,t) + \sum_t\sum_w x(i,w,t) + \sum_b\sum_j x(b,j,t) = 1, \forall t \quad (5)$$
$$\sum_t y(i,n,t) = 1, \forall i, \forall n \in N(i) \quad (6)$$
$$\sum_t p(b,n,t) = 1, \forall b, \forall n \in Q(b) \quad (7)$$
$$\sum_t z(j,n,t) = 1, \forall j, \forall n \in M(j) \quad (8)$$
$$\sum_t q(w,n,t) = 1, \forall w, \forall n \in R(w) \quad (9)$$
$$\sum_t y(i,n-1,t)\times t \le \sum_t y(i,n,t)\times t, \forall i, n = 2,3,..,N(i) \quad (10)$$
$$\sum_t p(b,n-1,t)\times t \le \sum_t p(b,n,t)\times t, \forall b, n = 2,3,..,Q(b) \quad (11)$$
$$\sum_t z(j,n-1,t)\times t \le \sum_t z(j,n,t)\times t, \forall j, n = 2,3,..,M(j) \quad (12)$$
$$\sum_t q(w,n-1,t)\times t \le \sum_t q(w,n,t)\times t, \forall w, n = 2,3,..,R(w) \quad (13)$$
$$\sum_i\sum_{n\in N(i)} y(i,j,t) + \sum_b\sum_{n\in Q(b)} p(b,n,t) = 1, \forall t \quad (14)$$
$$\sum_j\sum_{n\in M(j)} z(j,n,t) + \sum_w\sum_{n\in R(w)} q(w,n,t) = 1, \forall t \quad (15)$$
$$\sum_j x(i,j,t) + \sum_w x(i,w,t) = \sum_{n\in N(i)} y(i,n,t), \forall t, \forall i \quad (16)$$
$$\sum_j x(b,j,t) = \sum_{n\in Q(b)} p(b,n,t), \forall t, \forall b \quad (17)$$
$$\sum_i x(i,j,t) + \sum_b x(b,j,t) = \sum_{n\in M(j)} z(j,n,t), \forall t, \forall j \quad (18)$$
$$\sum_i x(i,w,t) = \sum_{n\in R(w)} p(w,n,t), \forall t, \forall w \quad (19)$$
$$ax(j,t) \le \sum_i x(i,j,t) + \sum_b x(b,j,t) + \frac{1}{3}\left(\sum_i\sum_{n\in N(i)} y(i,n,t)\times N(i,n) + \sum_j\sum_{n\in M(j)} z(j,n,t)\times M(j,n)\right) + \frac{1}{6}, \forall j, \forall t \quad (20)$$

$$ax(j,t) \geq \sum_i x(i,j,t) + \sum_b x(b,j,t) + \frac{1}{3}\left(\sum_i \sum_{n \in N(i)} y(i,n,t) \times N(i,n) + \sum_j \sum_{n \in M(j)} z(j,n,t) \times M(j,n)\right) - \frac{5}{6}, \forall j, \forall t \qquad (21)$$

Required number of rock-earth block to achieve the planned elevation is constrained by Eqs. (1) and (2). Maximum capacity of borrow (or waste) pit is constrained by Eqs. (3) and (4). The transportation of the rock-earth block is constrained by Eqs. (5) to (9). The excavating order of cut rock-earths from a cut pit (i) and that from a borrow pit are constrained by Eqs. (10) and (11), respectively. The backfilling order of fill rock-earths to a fill pit (j) and that to a disposal pit (w) are constrained by Eqs. (12) and (13), respectively. The index of a rock-earth in a cut pit (or borrow pit) and that in a fill pit (or disposal pit) are constrained by Eqs. (14) to (19). Moving a rock-earth which is nonconforming to a fill rock-earth is prohibited by the constraints shown in Eqs. (20) to (21).

III. Case Study

The earthmoving project shown in Fig. 1 was reproduced from existing studies (i.e., [6]) to verify the usability of the OPS method in the context of a land clearing earthwork. The earthwork consists of rough grading on a small office building site. The land size, earthwork volume, grid (or pit) spacing, and total number of grids (pits) are 90 m× 105 m, 43,770 m3, 15 m× 15 m, and 42 pits (=6× 7), respectively. The volume of a rock-earth block is set to 450 m3 (=length (15 m) × width (15 m) × depth (2 m)). The cut and/or fill volume of each pit required to accomplish the planned ground level are computed as shown in Table 1 using the current and planned elevations, which are denoted by dotted and solid lines, respectively.

A total of 96 rock-earth blocks are moved from cut pits to their corresponding fill pits. The optimal rock-earth types and the number of rock-earth blocks to move from a cut pit to its corresponding fill pit and the most economical cut-fill rock-earth block pairs satisfying the quality requirements of their fill pits are computed. However, the results are not presented due to lack of space. The total earthmoving cost is $11,111.02. The corrective action cost incurred by nonconforming rock-earths is $0 because those rock-earth blocks which do not conform to the fill rock-earth blocks are moved to their corresponding fill pits.

To minimize the earthmoving cost, the cut rock-earths in cut pits should be moved to the fill rock-earths of the nearest fill pits (i.e., 2, 5, 9, 12, 17, 24, 27, 32, 34, 39, and 42.). The optimal cut-fill pairs are denoted by the dash red lines; the excavator's repositioning sequences are [3 →10 →11 →18 →19 → 11→ 25→ 26→ 33 →40 →41] as denoted by the straight blue lines in Fig. 2.

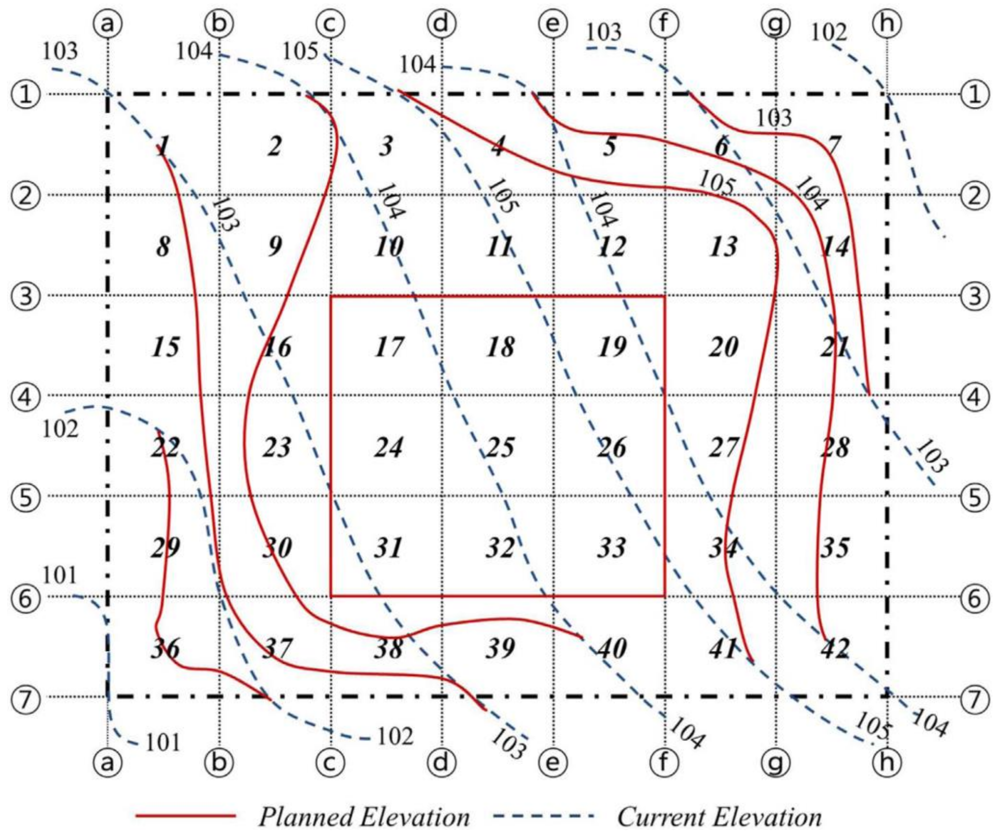

Figure 1. Earthwork job site grid plan

Table 1. Cut and fill worksheet

Pit ID	Cut amount(m³)			Fill amount(m³)			Pit ID	Cut amount(m³)			Fill amount(m³)		
	n_i=0	n_i=1	CV_i	m_j=0	m_j=1	FV_j		n_i=0	n_i=1	CV_i	m_j=0	m_j=1	FV_j
1	0	0	0	0	0	0	22	0	0	0	450	550	1,000
2	0	0	0	0	188	188	23	0	0	0	1,350	463	1,813
3	0	166	166	0	0	0	24	0	0	0	350	900	1,250
4	0	0	0	0	0	0	25	0	546	546	0	0	0
5	0	0	0	450	800	1,250	26	525	600	1,125	0	0	0
6	0	0	0	900	850	1,750	27	0	0	0	500	706	1,206
7	0	0	0	450	488	938	28	0	0	0	536	1000	1,563
8	0	0	0	313	0	313	29	0	0	0	688	500	1,188
9	0	0	0	450	300	750	30	0	0	0	2,000	500	2,500
10	157	0	157	0	0	0	31	0	0	0	1,500	375	1,875
11	0	597	597	0	0	0	32	0	0	0	206	0	206
12	0	0	0	417	450	867	33	800	388	1,188	0	0	0
13	0	0	0	2,250	875	3,125	34	0	0	0	0	48	48
14	0	0	0	900	1,163	2,063	35	0	0	0	0	875	875
15	0	0	0	0	750	750	36	0	0	0	0	625	625
16	0	0	0	738	450	1,188	37	0	0	0	900	413	1,313
17	0	0	0	0	170	170	38	0	0	0	613	450	1,063
18	0	787	787	0	0	0	39	0	0	0	209	0	209
19	0	773	773	0	0	0	40	0	563	563	0	0	0
20	0	0	0	1,350	1,025	2,375	41	0	128	128	0	0	0
21	0	0	0	2,125	0	2,125	42	0	0	0	0	313	313

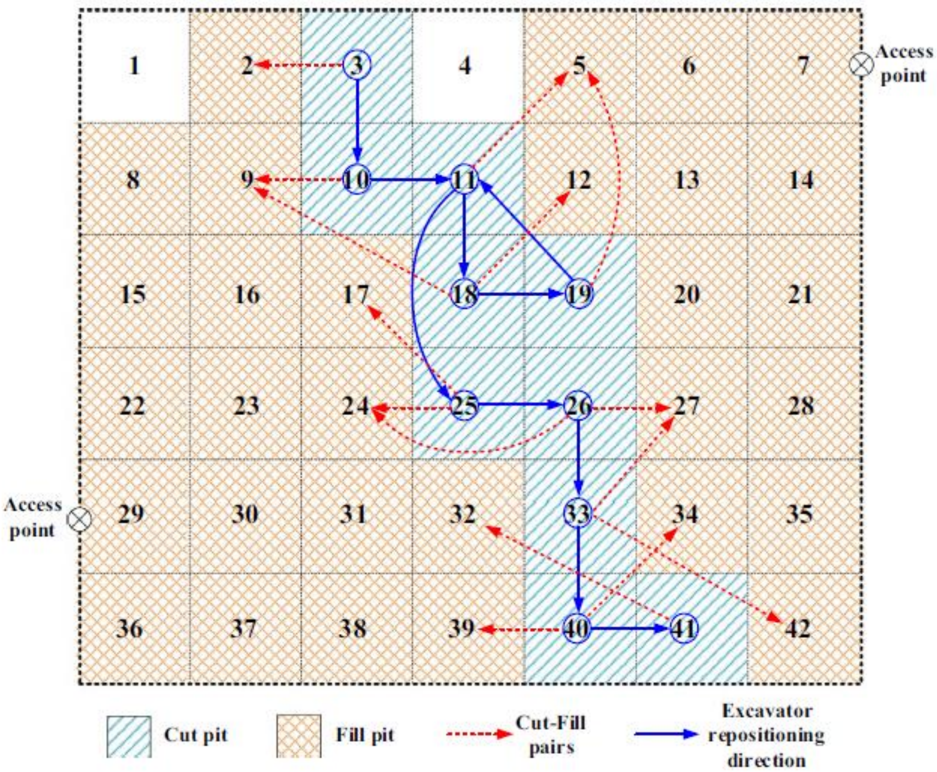

Figure 2. Graphical representation of the solution.

IV. Conclusion

This paper presents a MILP model that determines the optimal sets of cut-fill pairs and their sequence for EAP by taking into account the operational constraints. It minimizes (1) the total cost of moving the rock-earth blocks among the cut, fill, disposal, and borrow pits, (2) the additional expenses for correcting nonconforming rock-earth blocks, and (3) the excavators' repositioning cost. With the mathematical formula, an earthwork project manager may perform earthwork allocation planning by identifying the optimal cut-fill pairs and their sequence before and during the earthwork operation.

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