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WEAKLY b-δ OPEN FUNCTIONS

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Abstract: *In this paper, we introduce and study new classes of functions called b-δ-open functions and weakly b-δ-open functions by using the notions of b-δ-open sets and b-δ-closed sets. Some of its basic properties of these functions are investigated.*

Keywords: b-open set, δ-open set, b-δ-open set, weakly b-δ-open function, weakly-b-δ-closed function.

1. INTRODUCTION

The notions of δ-open sets, δ-closed set where introduced by Velicko [11] for the purpose of studying the important class of H-closed spaces. 1996, Andrijević [3] introduced a new class of generalized open sets called b-open sets in a topological space. This class is a subset of the class of β-open sets [1]. Also the class of b-open sets is a superset of the class of semi- open sets [5] and the class of preopen sets [6]. The purpose of this paper is to introduce and investigate the notions of weakly b-δ-open functions and weakly b-δ-closed functions. We investigate some of the fundamental properties of this class of functions. We recall some basic definitions and known results. Throughout the paper, X and Y (or (X, τ) and (Y, σ)) stand for topological spaces with no separation axioms assumed unless otherwise stated. Let A be a subset of X. The closure of A and the interior of A will be denoted by cl(A) and int(A), respectively.

2. PRELIMINARY

Definition 2.1. A subset A of a space X is said to be b-open [3] if $A \subseteq cl(int(A)) \cup int(cl(A))$. The complement of a b-open set is said to be b-closed. The intersection of all b-closed sets containing $A \subseteq X$ is called the b-closure of A and shall be denoted by bcl(A). The union of all b-open sets of X contained in A is called the b-interior of A and is denoted by bint(A). A subset A is said to be b-regular if it is b-open and b-closed. The family of all b-open (resp. b-closed, b-regular) subsets of a space X is denoted by BO(X) (resp. BC(X), BR(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by BO(X, x). The sets BC(X, x) and BR(X, x) are defined analogously.

Definition 2.2. A point $x \in X$ is called a δ-cluster [11] point of A if $int(cl(U)) \cap A \neq \varphi$ for every open set U of X containing x.

The set of all δ-cluster points of A is called the δ-closure of A and is denoted by δ-cl(A)). A subset A is said to be δ-closed if δ-cl(A) = A. The complement of a δ-closed set is said to be δ-open. The δ-interior of A is defined by the union of all δ-open sets contained in A and is denoted by δ-int(A)).

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Definition 2.3. A point $x \in X$ is called a b-δ-cluster [8] point of A if $int(bcl(U)) \cap A \neq \varphi$ for every b-open set U of X containing x. The set of all b-δ-cluster points of A is called the b-δ-closure of A and is denoted by b-δ-cl(A)). A subset A is said to be b-δ-closed if b-δ-cl(A) = A. The complement of a b-δ-closed set is said to be b-δ-open. The b-δ-interior of A is defined by the union of all b-δ-open sets contained in A and is denoted by b-δ-int(A)). The family of all b-δ-open (resp. b-δ-closed) sets of a space X is denoted by $B\delta O(X, \tau)$ (resp. $B\delta C(X, \tau)$).

Definition 2.4. A subset A of a space X is said to be α-open [7] (resp. semi-open [5], preopen[6], β-open[1] or semi-preopen [2]) if $A \subseteq int(cl(int(A)))$ (resp. $A \subseteq cl(int(A))$, $A \subseteq int(cl(A))$, $A \subseteq cl(int(cl(A)))$.

Definition 2.5. [4] $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly continuous if for every subset A of (X, τ), $f(cl(A)) \subseteq f(A)$.

Definition 2.6. [6] $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be pre-continuous if $f^{-1}(V)$ is pre-open in (X, τ) for every open set V of (Y, σ).

Definition 2.7. [1] $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be β-open if the image of each open set U of (X, τ) is a β-open set.

Lemma 2.5. [3] For a subset A of a space X, the following properties hold:
(1) $bint(A) = sint(A) \cup pint(A)$;
(2) $bcl(A) = scl(A) \cap pcl(A)$;
(3) $bcl(X - A) = X - bint(A)$;
(4) $x \in bcl(A)$ if and only if $A \cap U = \varphi$ for every $U \in BO(X, x)$;
(5) $A \in BC(X)$ if and only if $A = bcl(A)$;
(6) $pint(bcl(A)) = bcl(pint(A))$.

Lemma 2.6. [2] For a subset A of a space X, the following properties are hold:
(1) $\alpha int(A) = A \cap int(cl(int(A)))$;
(2) $sint(A) = A \cap cl(int(A))$;
(3) $pint(A) = A \cap int(cl(A))$.

3. WEAKLY b-δ OPEN FUNCTIONS

Definition 3.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be b-δ-open if for each open set U of (X, τ), $f(U)$ is b-δ-open.

Definition 3.2. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be weakly b-δ-open if $f(U) \subseteq$ b-δ-int(f(cl(U))) for each open set U of (X, τ).

Theorem 3.3. For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following conditions are equivalent:
(1) f is weakly b-δ-open,
(2) $f(\delta\text{-int}(A)) \subseteq$ b-δ-int(f(A)) for every subset of A of (X, τ),
(3) $\delta\text{-int}(f^{-1}(B)) \subseteq f^{-1}$(b-$\delta$-int(B)) for every subset of B of (Y, σ),
(4) f^{-1}(b-δ-cl(B) $\subseteq \delta\text{-cl}(f^{-1}(B))$ for every subset of B of (Y, σ).

Proof. (1)⇒(2): Let A be any subset of (X, τ) and $x \in \delta\text{-int}(A)$. Then there exists an open set U such that $x \in U \subseteq cl(U) \subseteq A$. Then, $f(x) \in f(U) \subseteq f(cl(U)) \subseteq f(A)$. Since f is weakly b-$\delta$-open, $f(U) \subseteq$ b-δ-int(f(cl(U))) $\subseteq$ b-δ-int(f(A)). This implies that $f(x) \in$ b-δ-int(f(A)). This shows that $x \in f^{-1}$(b-δ-int(f(A))). Thus $\delta\text{-int}(A) \subseteq f^{-1}$(b-$\delta$-int(f(A))) and so $f(\delta\text{-int}(A)) \subseteq$ b-δ-int(f(A)).

(2)⇒(3): Let B be any subset of (Y, σ). Then by (2), $f(\delta\text{-int}(f^{-1}(B))) \subseteq$ b-δ-int($f(f^{-1}(B))$) $\subseteq$ b-δ-int(B). Therefore $\delta\text{-int}(f^{-1}(B) \subseteq f^{-1}$(b-$\delta$-int(B)).

(3)⇒(4): Let B be any subset of (Y, σ). Using (3), we have $X - \delta\text{-cl}(f^{-1}(B)) = \delta\text{-int}(X - f^{-1}(B)) = \delta\text{-int}(f^{-1}(Y - B)) \subseteq f^{-1}$(b-$\delta$-int(Y − B)) = f^{-1}(Y − b-δ-cl(B)) = $X - f^{-1}$(b-δ-cl(B)). Therefore we obtain f^{-1}(b-δ-cl(B)) $\subseteq \delta\text{-cl}(f^{-1}(B))$.

(4)⇒(1): Let V be any open set of (X, τ) and $B = Y - f(cl(V))$. By (4),

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f^{-1}(b-δ-cl (Y − f(cl(V)))) ⊆ δ-cl(f^{-1}(Y − f(cl(V)))).Therefore, we obtain f^{-1}(Y −b-δ-int (f(cl(V)))) ⊆ δ-cl (X − f^{-1}(f(cl(V)))) ⊆ δ-cl (X −cl(V)).Hence V ⊆ δ-int (cl(V)) ⊆ f^{-1}(b-δ-int (f(cl(V)))) and f(V) ⊆ b-δ-int(f(cl(V))). This shows that f is weakly b-δ -open.

Theorem 3.4. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

(1) f is weakly b-δ -open;

(2) For each x ∈ X and each open subset U of (X, τ) containing x, there exists a b- δ-open set V containing f(x) such that V ⊆ f(cl(U)).

Proof. (1)⇒(2): Let x ∈ X and U be an open set in (X, τ) with x ∈ U. Since f is weakly b-δ-open, f(x) ∈ f(U) ⊆ b-δ-int(f(cl(U))). Let V = b-δ-int (f(cl(U))). Then V is b-δ-open and f(x) ∈ V ⊆ f(cl(U)).

(2)⇒(1): Let U be an open set in (X, τ) and let y ∈ f(U). It follows from (2) that V ⊆ f(cl (U)) for some b-δ-open set V in (Y, σ) containing y. Hence, we have y ∈ V ⊆ b-δ-int(f(cl(U))).This shows that f(U) ⊆ b-δ-int(f(cl(U))).Thus f is weakly b-δ-open.

Theorem 3.5. For a bijective function f: (X, τ) → (Y, σ) , the following conditions are equivalent:

(1) f is weakly b-δ-open,

(2) b-δ-cl(f(int(F))) ⊆ f(F) for each closed set F in (X, τ),

(3) b-δ-cl(f(U))) ⊆ f(cl(U)) for each open set U in (X, τ).

Proof. (1)⇒(2): Let F be a closed set in (X, τ). Then since f is weakly b-δ-open,
f(X − F)= ⊆ b-δ-int (f(cl(X −F)))= b-δ-int (f(cl(X −F))) and so Y −f(F) ⊆ Y −b-δ-cl (f(int(F))). Hence b-δ-cl (f(int(F))) ⊆ f(F) .

(2)⇒(3): Let U be an open set in (X, τ). Since cl(U) is a closed set and U ⊆ int(cl(U)), by (2), we have b-δ-cl(f(U)) ⊆ b-δ-cl(f(int(cl(U)))) ⊆ f(cl(U)).

(3)⇒(1): Let V be an open set of(X, τ).. Then we have Y - b-δ-int(f(cl(V))) = b-δ-cl(Y – f(cl(V))) = b-δ-cl(f(X – cl(V))) ⊆ f(cl(X – cl(V))) = f(X – int(cl(V))) ⊆f(X – V) = Y – f(V).Therefore, we have f(V) ⊆ b-δ-int(f(cl(V))) and hence f is weakly b-δ-open.

Theorem 3.6. For a function f : (X, τ) → (Y, σ), the following conditions are equivalent:

(1) f is weakly b-δ open;

(2) f(U) ⊆ b-δ-int(f(cl(U))) for each preopen set U of (X, τ),

(3) f(U) ⊆ b-δ-int(f(cl(U))) for each α -open set U of (X, τ),

(4) f(int(cl(U))) ⊆ b-δ-int(f(cl(U))) for each open set U of (X, τ),

(5) f(int(F)) ⊆ b-δ-int(f(F)) for each closed set F of (X, τ).

Proof: Follows from definitions of open, pre-open, α -open sets.

Theorem 3.7. Let X be a regular space. A function f: (X, τ) → (Y, σ) is weakly b-δ -open if and only if f is b-δ -open.
Proof. The sufficiency is clear.
For the necessity, let W be a nonempty open subset of (X, τ). For each x in W, let U_x be an open set such that x ∈ U_x ⊆ cl(U_x) ⊆ W. Hence we obtain that W = ∪ {U_x : x ∈ W} ⊆ ∪ {cl(U_x) : x ∈ W} and f(W) = ∪ {f(U_x) : x ∈ W} ⊆ ∪ {b-δ-int (f(cl(U_x))) : x ∈ W} ⊆b-δ-int (f(∪ {cl(U_x) : x ∈ W})) = b-δ-int (f(W)). Thus f is b-δ -open.

Theorem 3.8. If f: (X, τ)→ (Y, σ) is weakly b-δ -open and strongly continuous, then f is b-δ -open.
Proof. Let U be an open subset of (X, τ). Since f is weakly b-δ -open, f(U) ⊆ b-δ-int (f(cl(U))).However, because f is strongly continuous, f(U) ⊆ b-δ-int (f(U)). Therefore f(U) is b-δ -open.
Theorem 2.22. If a function f: (X, τ)→ (Y, σ) is weakly b-δ -open and precontinuous , then f is β-open.
Proof. Let U be an open subset of X . Then by weak b-δ -openness of f, f(U) ⊆ b-δ-int(f(cl(U))). Since f is precontinuous, f(cl(U)) ⊆ cl(f(U)).
Hence we obtain that

$$\begin{aligned} f(U) &\subseteq b\text{-}\delta\text{-int}\,(f(cl(U))) \\ &\subseteq b\text{-}\delta\text{-int}\,(cl(f(U))) \\ &= bint(cl(f(U))) \\ &= sint(cl(f(U))) \cup pint(cl(f(U))) \end{aligned}$$

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$$\subseteq cl(int(cl(f(U)))) \cup int(cl(f(U)))$$
$$\subseteq cl(int(cl(f(U))))$$

which shows that $f(U)$ is a β-open set in Y. Thus f is a β-open function.

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