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Portfolio Graph: Risk Vs. Return Trade Off

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Abstract: Due to possibility of high return, investment in stock is a challenging task for research community not only in finance but also in other disciplines of applied science and engineering. In this paper graph theory is used for portfolio optimization. All the constraints like upper bound of investment risk, fund constraint, time horizon etc. is being satisfied within this technique. The newly proposed technique outperforms the traditional techniques. However, due to paucity of time and non-availability of data set, the entire study is based on empirical in Indian context only. Future researchers can extend and verify its applicability for stock markets of other countries.

Keywords: Portfolio, Optimization, Investment Risk, Constraint Satisfaction Problem.

I. INTRODUCTION

A portfolio refers to a collection of securities held by an investor. Selection of securities within a portfolio obviously depends on risk aversion of the investor. So a good mix of securities implies different meaning to different people. Risk analysts usually consider return appreciation of securities pairwise within a portfolio. Types of securities included in a portfolio, may not necessarily be those which are positively correlated to each other [3]. The rational of such selection is to protect against adverse performance by limiting the risk. For example, to hedge against adverse downside risk of stock market, investors can simultaneously hold a put option to offset potential losses. This paper will address such issues using a graph theoretical framework. To the best of the knowledge, authors claim that sufficient literature is presently not available to link graph theory and portfolio management.

The next section describes graph theoretical concept followed by graph modeling of portfolio. Subsequently proposed methodology is illustrated followed by empirical evidence in Indian context. The paper terminates with vital conclusions along with scope future expansion.

II. GRAPH TERMINOLOGIES

A graph G is mathematically defined as tuple (V, E, φ) where V is a non-empty set whose elements are called vertices of G, E is a set whose elements are called edges of the graphs and φ is a function from E to V x V and called incidence function of the graph [2]. Geometrically the graph G = ({v1, v2, v3}, {e1, e2}, {(e1, v1, v2), (e2, v1, v2)} can be represented as in Fig. 1. Extending the notion of a graph, a signed graph is simply a graph where each edge corresponds to either positive or negative sign. For example Fig. 2 is a signed graph. Signed graph is relevant in describing the concept of structural balance [4]. According to the principle extolled by balance theory, a structure is balanced or stable iff all the cycles (closed walk between vertices and edges) of the signed graph are positive. Otherwise, it will be unbalanced. A cycle is said to be positive iff it has even number of negative edges in the cycle. Otherwise it will be negative. According to balance theory, any real life scenario with unbalanced structure has a tendency always to

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undergo changes, (i.e. correction in bias) in order to restore its state of balance. Several approaches for determining the degree of balance associated with signed graph has been reported [5]. Among them, the triangular degree of balance index is perhaps the most elegant. The index is calculated by taking the ratio of the number of positive triangles (i.e. 3 vertices cycle) to the total number of triangles in the signed graph.

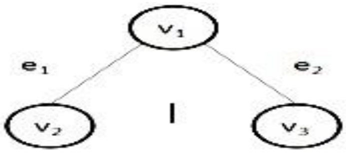

Fig. 1. A simple graph

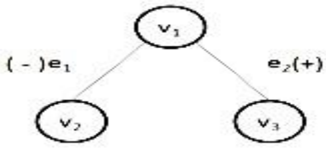

Fig. 2. A signed graph

This paper requires the concept of complete graph also. G is said to be complete iff it includes all possible edges between its vertices. A complete graph with n vertices is denoted by K_n as in Fig. 3.

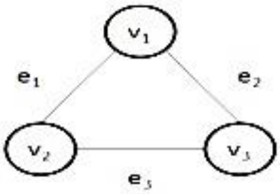

Fig. 3. A complete graph with 3 vertices (k_3)

If we denote negative edges by dotted line then some possibilities of complete graphs with 3, 4 and 5 vertices are shown in Fig. 4, Fig. 5 and Fig. 6 respectively.

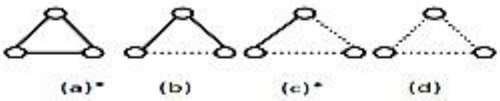

Fig. 4. Possible K_3

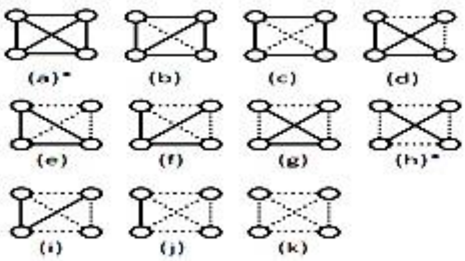

Fig. 5. Possible K_4

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Fig. 5. Possible K_4

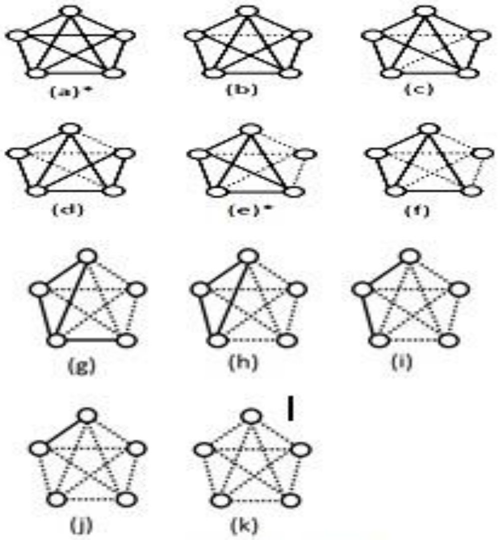

Fig. 6. Possible K_5

With these concepts of graph theory, now we are in a position to move in the next section describing link between graph theory and portfolio management.

III METHODOLOGY AND ANALYSIS

A portfolio of n securities can be represented as a complete graph whose vertices are securities and edges are sign of correlation between vertices. Hence a negative edge implies purpose of hedging. By analysing such portfolio graph, this paper recommends following benefits.

- The nature of portfolio graph can be used to judge the motive of investor. If protection against market shocks is the intention then graph should be balanced with at least one negative edge. On the other hand a balanced graph with all positive edges offers tendency to move either upside or downside. Hence it will be speculative in nature.
- A given portfolio graph is able to suggest possible course of actions for investors. If the portfolio graph is not balanced then suggestive restructuring can enhance the state of balance. On the other hand, for a given balanced graph, we can suggest structural transformations to move towards either hedging or speculation subject to risk.

Let us consider some possibilities of the portfolio graphs of 3, 4 and 5 vertices respectively as shown earlier in Fig. 4, Fig. 5 and Fig. 6. Only the balanced graphs are marked asterisk. Now we will discuss two possible situations. Case -I depicts the portfolio is balanced initially and case – II considers when it is unbalanced initially.
Case –I: Evidently 5(a) is balanced and has all positive edges and hence unprotected against downside risk. In order to hedge, the paper finds two possible courses of actions (recommended for an investor):

- Replacement of one or more vertices within Portfolio graph.
- Removal or Incorporation of one or more vertices within portfolio graph.

Evidently 5(e) and 5(h) both graphs are balanced. A transformation from 5(a) to 5(h) will involve replacing more than one security while 5(e) will need only one replacement.
A balanced K_n can be transformed to either balanced K_{n+m} or balanced K_{n-m} for some suitable natural number m. For example hedging can be done by incorporating stock option or future. It will imply transformation like 5(a) to 6(e).
Case –II: In this case, also we have two courses of actions as like as first case. To illustrate restructuring of unbalanced portfolio, we consider 6(a) which is unbalanced which can be transformed to balanced 6(e) without changing number of vertices. Even if a stable structure like 6(e) is not achievable, in order to reduce level of risk, investors may seek to restructure to one that has a higher degree of balance such as 6(h) or 6(g). Similarly to case–I, we can also restructure unbalanced portfolio by removal or incorporation of securities.
This section finds the evidence that goodness of a portfolio can be determined by its degree of balance. To arrive such empirical findings, we have considered 4 portfolios, each of which consists of 10 randomly selected stocks from National Stock Exchange of India, such that no two within the same portfolio, belong to same sector. The list of stocks is given in sorted order in Table 1.

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Table 1. Four Portfolios (Ten stocks each)

No	Stock Name (Industry Affiliation)-Portfolio1	Stock Name (Industry Affiliation)-Portfolio2	Stock Name (Industry Affiliation)-Portfolio3	Stock Name (Industry Affiliation)-Portfolio4
1	Axis Bank Ltd (Finance)	ABB LTD (Capital Goods)	ACC LTD (Housing Related)	Axis Bank Ltd (Finance)
2	BPCL (Refineries)	Adani Power Ltd (Power)	Axis Bank Ltd (Finance)	Bajaj Hindustan Ltd. (Agriculture)
3	CESC Ltd (Power)	Berger Paints India Ltd (Chemical & Petrochemical)	Bilcare Ltd (Health care)	Berger Paints India Ltd (Chemical & Petrochemical)
4	CMC Ltd (Software)	CIPLA (Pharmaceutical)	CIPLA (Pharmaceutical)	Bharat Forge Ltd (Transport Equipments)
5	DLF (Housing Related)	Dish TV India Ltd (Media& Publishing)	DLF (Housing Related)	DLF (Housing Related)
6	EIH Ltd (Tourism)	EIH Ltd (Tourism)	Escorts Ltd (Transport Equipments)	EIH Ltd (Tourism)
7	FDC Ltd (Healthcare)	Emami Ltd (FMCG)	Finolex Industries Ltd (Chemical & Petrochemical)	Entertainment Network(India) Ltd (Media & Publishing)
8	Grasim Industries Ltd (Textile)	Grasim Industries Ltd (Textile)	Grasim Industries Ltd (Textile)	Essar Oil Ltd (Refineries)
9	Havel's India Ltd (Capital Goods)	HCL Technologies Ltd (Software)	GTL Infrastructure Ltd (Telecom)	Havel's India Ltd (Capital Goods)
10	ITC Ltd (FMCG)	Hindustan Unilever Ltd (Diversified)	Hindustan Unilever Ltd (Diversified)	Hindalco Industries Ltd (Aluminum)

Source of Industry Affiliation: NSE India

The sample is drawn with replacement between portfolios but without replacement within a portfolio. Naturally, the same stock may participate in different portfolio and restrict our choice in such a way that no two portfolios have more than 5 stocks (50% of 10 stocks) in common. The merit of such selection is not to be biased with predefined sectorial correlation between stocks (as nicely pointed out by [1]. Depending on their average prices (i.e. (closing prices + opening prices)/2) from 01.04.10 to 30.11.14, we have completed pair wise correlation$\rho = \frac{Cov(x,y)}{\sigma x\, \sigma y}$ using statistical package SPSS and use the sign of ρ in our empirical study. Now for each of these portfolios, we constructed their portfolio graphs and recorded their balance index shown in Table 2.

Table 2. Rank of four Portfolios (Ten stocks each) vs. Balanced index of their graph.

Portfolio	Balance Index	Sharpe	Treyner	Jenson	Rank
1	0.314	-0.0089	-0.1370	-0.0640	**4**
2	0.726	0.0041	0.2056	0.2957	**1**
3	0.568	0.0015	0.0527	0.1492	**3**
4	0.621	0.0023	0.1663	0.2141	**2**

Result: Computed.

It is evident from finance literature that only return cannot be the proper tool for measuring performance of any portfolio. One must consider the risk undertaken by each portfolio also. For measuring effectiveness of our portfolios, we have used Sharpe, Treyner, and Jenson which are the proven tools in financial management.
With these measures, we computed our 4 portfolios to rank them and results are presented in Table 2. From those results, it is clear that portfolio graph with highest degree of balance is proved to be rank first and similarly remaining portfolios are ranked according as decreasing balance of index. Though the author agree personally that to keep the degree of balance as the sole criteria to invest may not work fully but to significant extent this paper proved it viable for randomly selected portfolios from Indian stock market. It opens a new area of research towards stock market using a tool like graph theory which has almost never been used previously.

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I V CONCLUSION

Main objective of this paper is to illustrate a methodology describing how complete signed balanced graph representation of portfolio can be used for risk analysis. It also provides a surprisingly elegant method to describe state of balance for a given portfolio graph. The paper recommends a portfolio graph which is complete signed and balanced for effective hedging but market dynamics can anytime change these characteristics of an existing portfolio. Proactive measures to restructure the portfolio graph may therefore be a necessary challenge in finance literature (future scope). In the empirical part, this paper attempts to study the goodness of a portfolio in terms of its balance index. It has been found that more the balance index, more suitable the portfolio is. Hence the paper recommends the investors to go for that portfolio which has more degree of balance. Although our illustrations are based on simple portfolio but the idea is generally applicable for portfolio involving larger cluster of assets. As the number of assets increases, the geometrical representation of portfolio graph should provide attractive alternatives for exposing structurally weak portfolios. Hence finding optimal transformation (of portfolio graph structure) subject to the available investment capital constraint is another challenge in future. Such optimality of a restructuring can be perceived in terms of number of vertices (securities) involved, changing amount of investment capital etc.

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