Effects of higher order-dispersion on dissipative structures in a photonic crystal fiber resonator

L.BAHLOUL, L.CHERBI, H.HARIZ
Laboratory of Instrumentation, U S T H B,
Algies, Alegria.
Lyes_86@live.fr, cherbi_lynda@hotmail.com
haffidoff@gmail.com

M.TLIDI
Faculté des sciences U L B
Brucelles, Belgium.
mtlidi@ulb.ac.be

***Abstract*—we describe in this work the influence of higher-order of dispersion on dissipative structures generated in a photonic crystal fiber cavity. Numerical simulations have shown that the term B_4 influences the frequency of the signal while the term B_3 introduces a phase shift which results in a temporal constant velocity proportional to this term without changing the frequency.**

***Keywords-component; resonant cavity; Photonic Cristal fiber 'PCF'; modulation instability, nonlinear effects*.**

I. INTRODUCTION

Photonic crystal fibers (PCFs) can be defined as being 2-dimensional photonic crystal consisting of periodic setting air holes in the cladding of the fiber traversing along the axis of propagation. Such structures can have a reflectivity that varies as a function of the wavelength. [1]

There are two categories of PCF fibers determined using the propagation of light in the fiber. The total internal reflections PCF and the bandgap PCF.

The PCF have several characteristics that allow their use in many applications, among these characteristics the control of chromatic dispersion which is a serious problem in standard fibers, also we can design PCF with a high nonlinearity, that's why the use of photonic crystal fiber (PCF) has seen a significant rise due to all benefits that they can offer compared to standard ones, we can for example control the wavelength of zero dispersion and get more interesting values than those found with standard fiber, which allow exploring the higher order dispersion.

We will therefore use a PCF fiber as a medium to design a resonant cavity which has been the subject of several studies of researches[2,3,4,5].

The advantage of working in these cavities is to exploit the existence modulation instability [2, 3,4] "IM" which is defined as a balance between chromatic dispersion and nonlinear effects (Kerr effect) to generate specific signals as dissipative structures that are modulated signals or localized structures that are the result of interaction between the modulated solution and the homogeneous one. These last structures are also called localized cavity solitons.

These signals can be used in several applications like the generation of ultra-high bit rate pulses for data transmitting or in temporal storing of data with a high speed.

II. DESCRIPTION OF THE PCF CAVITY

Our system consists of a section of PCF fiber looped to itself using an optical coupler, which is characterized by a transmission coefficient T and a reflection one R.

The cavity is lunched continuously by optical power CW.

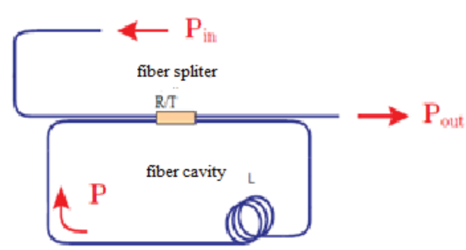

Figure 1: schematic description of the PCF resonant cavity

After each cavity round, there will be a portion of the optical power out of the cavity and the other part will be superposed with the Continuous power at the input.

III. MATHEMATICAL MODEL

The equation that governs the propagation of a light wave in the optical fiber is a nonlinear Schrodinger equation (NLS) which follows [7]:

$$\frac{\partial E}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 E}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 E}{\partial t^3} - j\frac{\beta_4}{2}\frac{\partial^4 E}{\partial t^4} = j\gamma|A|^2 E - \frac{\alpha}{2} \quad (1)$$

Where: E is the pulse envelope, (B_2, B_3, B_4) are dispersions of order 2, 3 and 4, γ: The coefficient of nonlinearity and α is the linear attenuation.

The equation (1) is then submitted to the boundary conditions of the cavity [2, 3] which means that the intracavity field "E" must vary little from one round of cavity to another and the phase shift between it and the CW pump should be as small as possible to ensure the resonance of our system.

These conditions allow us to perform an average over the intracavity field and subsequently establish the equation that governs the propagation inside the cavity under the following form [2, 3, 6]

$$\frac{\partial E}{\partial t} = S - (1 + i\Delta)E + i|E|^2 E - iB_2\frac{\partial^2 E}{\partial \tau^2} + B_3\frac{\partial^3 E}{\partial \tau^3} + iB_4\frac{\partial^4 E}{\partial \tau^4}$$

(2)

Where: S is the input power, E is the envelope of the intracavity field, the detuning Δ is the parameter which characterizes the phase shift between the pump and E, t is the time of a cavity round and τ is the time in a referential that depends on the group velocity of the wave.

The stationary and homogeneous solution Es of (2) satisfies:

$$S = (1 + i\Delta)E_s - i|E_s|^2 E_s \quad (3)$$

We now find the thresholds of onset of modulation instability (MI), to do this we disturb the homogeneous solution E_s with a disturbance that has the following form: a = exp (λt -iΩτ), we have then:

$$E = E_s + a \quad (4)$$

After that we inject (4) in (2) and we perform a linear stability analysis on our system. [2, 3, 8].

This study consists in taking the linear part of (2) and look for power thresholds for which the modulation instability appears and the frequency corresponding to each threshold.

The result of this study has shown the existence of two instability thresholds:

The first threshold is $I_{1m} = |E_{1m}|^2 = 1$, at this level we noticed the appearance of two frequencies at the same time, and their expressions are as follows [2]:

$$\Omega^2 = \frac{-B_2 \pm \sqrt{{B_2}^2 + 4(\Delta - 2)B_4}}{2B_4} \quad (5)$$

The second threshold is expressed as:

$$|E_{2m}|^2 = (2\Delta_{eff+\sqrt{\Delta_{eff-3}}})/3 \quad (6)$$

With: $\Delta_{eff} = \frac{{B_2}^2}{4B_4} + \Delta$.

At this level we find a single frequency, and it is expressed as follows:

$$\Omega^2 = \frac{-B_2}{2B_4} \quad (7)$$

These results are very important because they give us a precise information about the frequency of the signals which can be generated inside the cavity and also allow us to know which parameters of the fiber that influence these frequencies, we note that the term B_3 doesn't influence the frequency of these signals.

However, the linear stability analysis shows that the term B_3 introduces a phase noted K_c, which is expressed as:

$$K_c = B_3\Omega^3 \quad (8)$$

This phase will induce a temporal drift of the wave between a round of cavity and another; it is interpreted as constant velocity which is calculated as follows:

$$V_l = \frac{d\,K_c}{d\Omega} = \frac{dB_3\Omega^3}{d\Omega} = 3B_3\Omega^2 \quad (9)$$

IV. EFFECTS OF B_3 AND B_4 ON CAVITY SIGNALS

A. $B_3 = 0$

In this section we numerically integrate (2) near the first instability threshold while forcing the system to generate a single frequency instead of two as predicted by the preceding calculation, this is can be made by choosing the parameters of the fiber which ensure the following condition:

$$B_2{}^2 + 4(\Delta - 2)B_4 = 0 \quad (10)$$

The expression of the frequency becomes:

$$\Omega^2 = \frac{-B_2}{2B_4} \quad (11)$$

To ensure that Modulation Instability appears, we have as a first step to disturb the homogeneous solution Es as shown in the following figure:

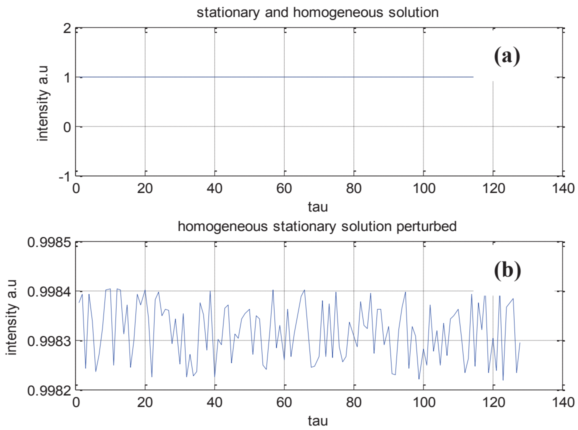

Figure 2: (a) homogeneous stationary solution (b) homogeneous stationary solution perturbed for the following parameters:

Δ=1.3, S=1.129, B_2=-0.75, B_3=0, B_4=0.2.

We see clearly from the “Fig. 2” that the homogeneous solution has experienced the phenomena of modulation instability, this is the reason of the appearance of a totally disturbed signal, we also note that the term B_3 was ignored so we can conclude in this case that the MI is a balance between the second and the forth order of dispersion on one side and the nonlinear effects (Kerr effect) on other side as predicted by analytical calculation.

After that, the perturbed signal will travel inside the cavity by doing several rounds which will overlap with the pump signal at the input of the cavity for each round, this will increase the non-linear effects and permits to obtain a dissipative structure as shown in the following figure:

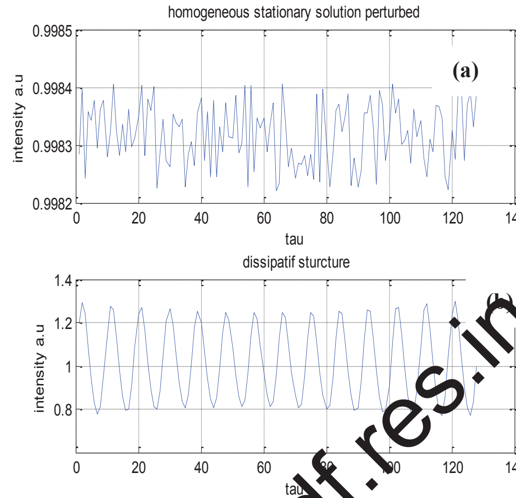

Figure 3: (a) perturbed homogeneous and stationary solution

(b): dissipative structure (modulated solution) for the same parameters of Figure 2.

So we can define the dissipative structures as the result of the saturation of the signal which was disturbed by the phenomena of modulation instability. This saturation is due to the accumulation of nonlinear effects after each round cavity.

B. $B_3 \neq 0$

We now repeat the previous simulation except that this time we will consider the influence of the term B_3 [3], we noticed that for the same parameters we find the same modulated signal but with a phase shift between two successive rounds of cavity as shown in this figure:

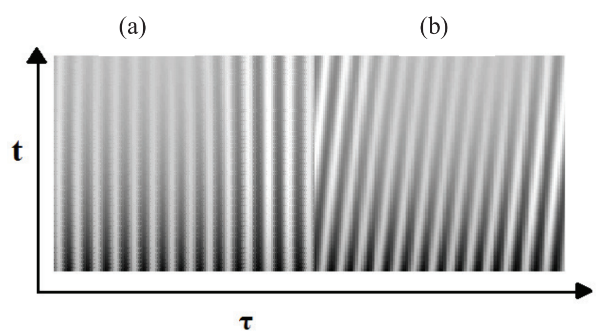

Figure 4: 3D map that shows the time evolution of the modulated intracavity solution with the following parameters:

Δ=1.3, S=1.129; B_2=-0.75, $B_4 = 0.2$

(a) Stationary solution B_3= 0.

(b) Moving solution B_3= 0.25.

"Fig.4" clearly shows the difference between the intracavity signal with or without the effect of B_3.

We first noticed that the two figures have exactly the same frequency with the same amplitude in the presence or absence of B_3, which confirms the predicted results in the analytical calculation where it is noted that B_3 does not influence the frequency of generated signal.

However, we see that in presence of B_3, it introduces a phase after each cavity round that gives a constant velocity to the wave during its evolution. The numerical calculation of the velocity is derived from the determination of the slope of the curve presented in Fig"4.b"

$$V \cong \frac{11}{39} = 0.282$$

However if we calculate the velocity with the analytical method (9) we find that $V_l = 1.4$.

It is obvious that there is a difference between the velocity calculated by the method of linear stability and the one found in our simulations. It is due to the presence of another velocity that occurs when we are near the threshold of MI; it is the nonlinear velocity and in turn affects the total velocity of the wave which becomes the sum of both.

To find the expression of this velocity, it is necessary to make a nonlinear calculation which will be the subject of the future papers.

V. CONCLUSION

We arrived in this work to generate in a PCF resonant cavity a train of pulses of ultra-high bit rate with a stable amplitude and frequency by ensuring the condition of onset of modulation instability.

Besides , by using a PCF fiber we were able to introduce the effects of higher-order dispersion, we have shown that the term of B_4 influenced the frequency of the generated signal while the term B_3 introduced a temporal phase shift after each cavity round, this phase is interpreted as a wave velocity that depends on B_3.

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