Fast Iterative algorithms for Airborne Radar Processing

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Abstract—**The conventional space-time adaptive processing (STAP) such as the sample matrix inversion (SMI) or the principal components (PC) methods are computationally costly and require the estimation of the clutter covariance matrix from secondary data, which are assumed to be independent and identically distributed. However, in monostatic airborne radar, the data are not stationary. Consequently, to circumvent such a problem, we propose to investigate the performances of adaptive recursive subspace-based algorithms of linear complexity using projection approximation subspace tracking (PAST) and orthonormal PAST(OPAST). In addition, we apply the fast implementation of the power iterations method for subspace tracking (FAPI), based on a less restrictive approximation than the well known projection approximation (API). Performance curves show that PAST, OPAST, API and FAPI algorithms do indeed allow a good detection of slow moving targets even with a low rank covariance matrix. We also show that in the case of Doppler ambiguous environment when combined a pseudo random staggered PRF, these algorithms give better results than the methods based on eigenvalues decomposition.**

Keywords-**Airborne Radar Processing; STAP.**

I. Introduction

For ground radars, all clutter echoes are received with a Doppler frequency zero, while for airborne radars, the total of all arrivals produces a Doppler clutter broadband. Space-time processing, STAP, can provide a rejection of clutter to detect slow targets. Typically, STAP means the simultaneous processing of both the spatial signals received by multiple elements of an array antenna and the temporal signals provided by the echoes from a coherent pulse interval (CPI) [1, 2]. A space-time clutter filter has a narrow clutter notch, so that even slow targets fall into the pass band. Brennan and Reed [3] first introduced STAP to the radar community in 1973. With the recent advancement of high speed, high performance digital signal processors, STAP is becoming an integral part of airborne or space-borne radars for MTI functions. However, the main disadvantage of STAP is its high computational cost, since it utilizes complex matrix operations and often in an iterative way. For this reason, some reduced-rank STAP algorithms have been developed [4-9]. In [1-14], it was shown that STAP has a good ability to extract targets from Doppler-spread clutter. On the other hand, the conventional fully adaptive STAP known as the SMI method as well as the subspace-based eigencanceller are not recommended due to their prohibitive computational cost, which makes their real-time implementation very difficult [1].

In [14], we studied the effect of the radar parameters on the detection of slow target where the point of Doppler ambiguities, reduction of rank and staggered PRF are well explained. In [15], we studied the performance of two iterative algorithms on the detection; namely PAST and OPAST algorithms. In this paper, we extent the analysis and study of subspace tracking for interference suppression detection algorithms of PAST and OPAST and include two others: the approximation power iteration (API) and its fast version (FAPI). The respective performances of these four detection algorithms are compared to the principal component method.

We will show that good performances are achieved even in an ambiguous environment when using a staggered PRF. In Section 2, the mathematical model of data to describe the environment in which the radar operates is presented. In Section 3, we give a brief description of STAP with reduced rank (PC method) and staggered PRF. The proposed iterative algorithms are given in Section 4. In Section 5, the results are presented and discussed. A conclusion is given in Section 6 highlighting the main results obtained.

II. Mathematical Model of Data

Consider a space time network with N antenna elements uniformly spaced and M delay elements for any antenna element at a constant pulse-repetition frequency (PRF). The data are then processed on one range of interest which corresponds to one slice of the data cube.

A space time snapshot at range k in the presence of a target is given by [1]

$$\boldsymbol{X} = \alpha \boldsymbol{S} + \boldsymbol{X_i} \tag{1}$$

where, $\boldsymbol{X_i}$ is the vector of interferences (noise, jamming and clutter), α is the target amplitude and $\boldsymbol{S}$ is the space-time steering vector given by $\boldsymbol{S} = \boldsymbol{S_t} \otimes \boldsymbol{S_s}$, $\boldsymbol{S_t}$ and $\boldsymbol{S_s}$ are the temporal and space vectors given respectively by

$$\boldsymbol{S_t} = [1; e^{-j2\pi \boldsymbol{F_t}}; e^{-j2\pi 2\boldsymbol{F_t}}; \ldots; e^{-j2\pi(\boldsymbol{M}-1)\boldsymbol{F_t}}] \tag{2}$$

$$S_s = [1; e^{-j2\pi F_s}; e^{-j2\pi 2F_s}; ...; e^{-j2\pi(N-1)F_s}] \quad (3)$$

$F_t = F_d / PRF$ and $F_s = d.\sin\theta/\lambda$ are, respectively, the normalized Doppler and spatial frequency; d is the distance between the antenna elements and θ is the azimuth angle. The optimum weight of the STAP, which maximizes the signal to interference noise ratio, SINR, is given by [1]

$$W_{opt} = \alpha R^{-1} S \quad (4)$$

R is the covariance matrix of the interferences, which is supposed to be known and is the sum of covariance matrices of the clutter, jammers and thermal noise. In practice, R is not known and must be estimated from the snapshots. The well-known SMI gives an estimate of the matrix by averaging over the secondary range cells, such that

$$\hat{R} = \frac{1}{L} \sum_{l=1, l \neq k}^{N} X_l X_l^H \quad (5)$$

where k is the test range cell, and L is the number of secondary range cells.

The performance of the processor can be discussed in terms of the Improvement Factor (IF). IF is defined as the ratio of the SINR of the output to that of the input of the Direct Form Processor (DFP) and is given by [1]

$$IF_{opt} = \frac{W^H S.S^H W.tr(R)}{W^H R.W.S^H S} \quad (6)$$

where W is the optimum weights of the interference plus noise rejection filter.

Note that a notch, which is a reversed peak of the clutter, appears at the frequency in the direction of sight of the radar, while the width of this notch gives a measurement of the detection of slow moving targets.

III. STAP WITH REDUCED RANK AND STAGGERED PRF

The fully adaptive techniques of signal processing cannot be applied for a real-time processing because of the high computational cost. The methods with reduced rank exploit the nature of the low rank matrix of interferences.

The idea is the separation of the overall space into an interference subspace and a noise subspace [4-6]. A common method to obtain these subspaces is via singular value decomposition (SVD) of the interference-plus-noise covariance matrix. Such methods can reduce the sample support requirement to *O(2r)* (number of operations), where *r* is the rank of the covariance matrix, but at the expense of a considerable computational complexity due to the SVD $O((NM)^3)$ which prevents the real-time applications.

The partially adaptive algorithms of the STAP consists in transforming the data with a matrix $V \in C^{MN \times r}$ where r << *MN* in order to reduce the computational time. There are several methods for the covariance matrix rank reduction [4-9], which may differ in the shape of the processor as well as in the selection of the columns of the matrix. The principal component is based on the eigenvectors conservation of the matrix of covariance of interferences corresponding to the dominant eigenvalues [4]. If we assume that the *r* columns of V are a subset of the eigenvectors of R, the improvement factor of the reduced rank can then be written as [2]

$$IF_{RR} = S^H V (V^H R V)^{-1} V^H S \frac{tr(V^H R V)}{S^H V.V^H S} \quad (7)$$

It is known that high PRF radars are ambiguous in range while for low PRF radars Doppler ambiguities occur and are caused by the overlapping of the edge lines with the true spectrum. This overlapping decrease gradually with each time the PRF is increased because the edge lines move away from each other by leaving the true spectrum without a shift. Therefore, the idea of using the change of PRF appeared to solve the problem of Doppler ambiguities. In [13], Klemm proposed the pseudorandom change of PRF consists in varying the interval of repetition of impulses PRI in a pseudo-random way by multiplying it by the term $(1+\varepsilon.r(m))$ for each impulse m where the random part r(m) is uniformly distributed on the interval [-1.1]. He demonstrated that this allows the elimination of Doppler ambiguities for optimum detection.

IV. ITERATIVE ALGORITHMS FOR STAP

To reduce the computational burden linked to SVD, recursive subspace tracking algorithms that update the subspace estimate, as long as a new snapshot is received, have been proposed in the literature [17-23]. They consist in recursively updating a weight vector at time *k* from the weight vectors obtained at time *k-1* and by taking into account the current snapshot. These algorithms generally involve less computational operations than their block counterparts. They showed their effectiveness in several domains of signal processing, and in particular in array signal processing, filtering, spectral analysis, prediction, and in many other applications such as channel equalization, noise cancellation, speech coding, etc. They offer interesting perspectives in STAP. They can be classified depending on their computational complexities into *O((NM)2r)*, $O(NMr^2)$, and *O(NMr)* operations at each iteration (update) [23].

In this context, we consider the class of the fastest, most robust and effective algorithms referred to as the linear or low complexity because they are the most important ones due to their suitability for real-time applications. We propose the application and the evaluation of the performances of the algorithm PAST and its orthogonal version (OPAST), then we apply the algorithms API and FAPI. A brief description of the main ideas of these algorithms is as follows.

The projection approximation subspace tracking algorithm (PAST) proposed by Yang [18] is based on a novel interpretation of the signal subspace as the solution of an unconstrained minimization task. Because of its efficiency and simplicity, it is one of the successful subspace tracking algorithms known of the class with linear complexity *O(NMr)*. It is based on the optimization of the following criterion

$$J(W) = E\left(\left\| x - WW^H.x \right\|^2\right) \tag{8}$$

where $W \in C^{NMxr}$ is the matrix which engender the dominant subspace. It was shown that $J(W)$ has global minimum unique reached for $W_{opt} = U_d Q$ where U_d contains the *r* greatest eigenvectors of R and $Q \in C^{rxr}$ is a unitary matrix [18].

A projection approximation is utilized to reduce the minimization task to the well known exponentially weighted least square problem. Substituting the mean in (8) by an exponential weighted sum, we obtain

$$J(W(t)) = \sum_{i=1}^{t} \mu^{t-i} \left\| x(i) - W(t)W^H(t).x(i) \right\|^2 \tag{9}$$

Recursive Least Square (RLS) methods are then used to track the signal subspace considering the following approximation

$$W^H(t)\, x(t) = W^H(t-1).x(t) \tag{10}$$

This approximation can be interpreted as follows: the projection of an observation vector ***x(t)*** on the columns of ***W(t)*** (unknown at the iteration ***t***) is almost equivalent at its projection on ***W (t−1)*** estimated before on the observation ***x(t)*** (thus its name "Approximation Projection"). Substituting (10) in (9), we obtain the following modified criterion [17]:

$$J(W(t)) = \sum_{i=1}^{t} \mu^{t-i} \left\| x(i) - W(t).y(i) \right\|^2 \tag{11}$$

where W is the estimated interference subspace basis, μ is the forgetting factor, $0 \le \mu < 1$, and $y(i) = W^H(i-1).x(t)$ is the cost function.

The minimization of this criterion is equivalent to the PAST form

$$W(t) = R(t)W(t-1)(W^H(t-1)R(t)W(t-1))^{-1} \tag{12}$$

PAST has fast convergence, because it is a recursive type of implementation. However, it does not guarantee orthonormality of the estimated subspace matrix, which might be needed in some applications [23]. To ensure global convergence and to guarantee the orthonormality of the noise subspace matrix at each iteration, in [19], an explicit orthonormalization at each iteration of PAST was carried out, which resulted in the orthonormal PAST (OPAST) algorithm.

The Approximated Power Iteration (API) and Fast Approximated Power Iteration (FAPI) algorithms derive from the power method [19]. A less restrictive approximation than for PAST is used. Indeed it concerns the projection on the estimated subspace instead of the estimated subspace itself:

$$W(t).W^H(t) \approx W(t-1).W^H(t-1) \tag{13}$$

Equation (13) is equivalent to

$$W(t) = W(t-1)\Theta(t) \tag{14}$$

where $\Theta(t) = W^H(t-1)W(t)$. Thus

$$W(t) = (W(t-1) + e(t)g^H(t))\Theta^H(t) \tag{15}$$

Using equation (15), knowing that $W(t-1)$ and $W(t)$ are orthonormal, in addition that the error vector ***e(t)*** is orthonormal to $W(t-1)$ then

$$\Theta(t) = \left(\sqrt{I_r + \left\| e(t) \right\|^2 g(t)g(t)^H}\right)^{-1/2} \tag{16}$$

FAPI algorithm is a fast implementation of API (*O(NMr)*) which consists in substituting $\Theta(t)$ by a faster computation of the inverse square root.

V. Results and Discussion

In this Section, we discuss the effect of some algorithms, based on the reduction of the rank of the covariance matrix, on the detection of a target with a low power (SNR=0dB) and with a slow speed. The simulated environment considered is a linear side-looking network of *N*=8 antennas apart with $d = \lambda/2$, half of the emitted wavelength, and the assumed number in the coherent processing cube is *M*=10. The dimension of the adaptive process is thus *MN* = 80. The elevation angle is fixed to 20°. The speed of the airborne radar is V_R=100m/s, and the frequency of transmission is 0.3GHz. The assumed environment of interferences consists of five jammers and ground clutter. The jammers are at azimuth angles of 0°, 180°, 60°, 90°, and 72°, with respective jammer to noise ratios (JNRs) of 13dB, 12dB, 11dB, 10dB and 9dB. The clutter to noise ratio (CNR) is set equal to 8dB. This clutter covers the band [30°, 30°]. All the simulations are carried for more than 20 Monte Carlo runs. For the iterative algorithms, we use the improvement factor given by Expression (6) where $W = (I - ww^H)S$ and w is the estimated subspace of interferences. The forgetting factor of PAST, OPAST, API and FAPI algorithms is fixed at $\mu = 0.99$.

Fig. 1 shows the eigenspectra for a known covariance matrix and for an SMI STAP covariance matrix. We note a clear distinction between the interferences subspace and noise subspace. An extension to Brenann's rule, for a number of an effective rank for the covariance matrix of a side-looking radar, has been derived recently in [9] and is given by *r =N+(β+J)(M−1)* [16], where *J* denotes the number of jammers. In this case *r* = 35 and can actually be read directly from Fig. 1.

The aim of this paper is to demonstrate the robustness of the algorithms PAST/ OPAST and FAPI by comparing their performances to PC method for low rank covariance matrix. On the other hand, we have to circumvent the problem of Doppler ambiguities by applying a staggered PRF method namely the pseudorandom one. We start by illustrating the effect of the rank and the PRF on the detection of slow targets.

Fig. 2 shows the effect of the constant PRF on the eigenspectra. We observe that for high values of the PRF (low values of β), the subspace of signal is spread and thus there is an increase of rank of the covariance matrix. This is due to the overlap of the sidebands which becomes more important with the decline in PRF, and consequently the effect of Doppler ambiguities will be more important and makes the target more difficult to detect. Indeed, if one compares the two results in (a) and (b) of Fig. 3 for the different PRFs; for a PRF equal the Nyquist frequency: PRF=4V_R/λ (sampling) and a PRF equal to half the frequency of Nyquist: PRF =2V_R/λ (subsampling), respectively, although we observe the well-known aliasing phenomenon, expressed here by the appearance of echoes sidelobe clutter. We can therefore say that the choice of the PRF is essential to ensure better detection.

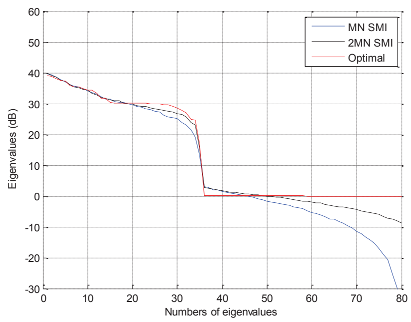

Figure 1. Eigenspectra for known covariance matrix and SMI STAP covariance matrix with N = 8, M = 10, J=2, JNR=CNR = 30 dB, β = 1 or $\boldsymbol{PRF} = 4\boldsymbol{V_R}/\lambda$

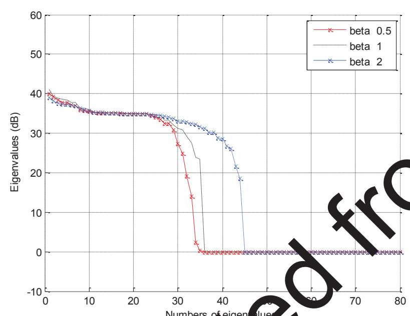

Figure 2. Effect of the PRF on the eigenspectra of the covariance matrix R, avec JNR=35 dB, N=8, M=10, CNR = 30 dB, J=2

Fig. 4 represents the reduction of rank using PC method in an unambiguous scenario and with different values of *r*. The results obtained are almost the same for *r*=40 and *r*=20, and the notch approaches the optimal processor. Thus, it is recommended to work with this last value to reduce the computational time. The clutter effect is more pronounced when *r* decreases and the detection of slow targets is no longer possible for low values (*r*=8). For a medium value of rank of the covariance matrix *r* = 20 and an unambiguous environment ($\boldsymbol{PRF} = 4\boldsymbol{V_R}/\lambda$), we can see from Fig. 5(a) that all the tested algorithms (PAST, OPAST, API and FAPI) are globally equivalent and present good performances: the notch is relatively thin compared to the optimal processor which leads to the detection of slow targets. We observe from Fig. 5(b) that recursive algorithms perform better in the case of a low rank value. Furthermore, we note that the recursive algorithms outperform the SMI algorithm in any case. For convenience and for more readability of the curves, The SMI STAP covariance matrix will not be shown on the next curves.

In the presence of ambiguities ($\boldsymbol{PRF} = 2\boldsymbol{V_R}/\lambda$), we see from Fig. 6, the appearance of ambiguous notches. This is again due to the aliasing phenomenon. Fig. 6(a) shows that iterative algorithms present acceptable detection performances than those of PC method. In Fig. 6(b), we observe that the detection becomes impossible for PC method. Contrarily, the use of recursive algorithms overcome this problem and improves the detection. Performances are comparable to that obtained by the optimal processor. In conclusion, we can see that adaptive algorithms give a much better performance than the PC method, in terms of detection when the radar operates in an unambiguous environment and with low rank of reduction.

To overcome the problem of ambiguities, we suggest the use of pseudorandom staggered PRF in combination of the iterative algorithms. We notice on the Fig. 7 that the detection becomes impossible with PC method while it is acceptable when applying iterative algorithms.

VI. CONCLUSION

In this paper we have considered the use of four iterative algorithms (PAST, OPAST, API and FAPI) for the suppression of interferences and thus, the detection of slow targets in monostatic airborne radar. We observed that the presence of ambiguities and the reduction of rank of the covariance matrix to low values degraded the performance of STAP in suppressing interferences and detecting slow targets. We showed that we can mitigate these problems by using recursive algorithms which can estimate recursively the weights of the clutter rejection filter. With a very low covariance matrix rank and with Doppler ambiguities, the simulation results confirmed the superior performance of the considered algorithms (PAST, OPAST, API and FAPI) compared to the SMI and PC methods even with the optimal filter. Also, it is shown that all the algorithms outperform SMI method when the covariance matrix is estimated from a data set with limited support.

This comparative study proved that iterative algorithms could be applied for the reduction of the rank for the STAP because they give similar performances as those given by the methods of rank reduction, but gave a much better performance for low rank values. In addition, they present a very low computational complexity. In fact, it can be viewed From Table 1, that the complexity burden is *O(MN)* instead of $O((MN)^3)$ for the PC processor. That's why these algorithms can be considered as an economical approach in comparison with the other techniques. In addition, it was proven that the problem of Doppler ambiguities is resolved too, and thus these algorithms (PAST, OPAST, API and FAPI) achieved good performance for the detection of slow targets even with a low rank and an ambiguous environment.

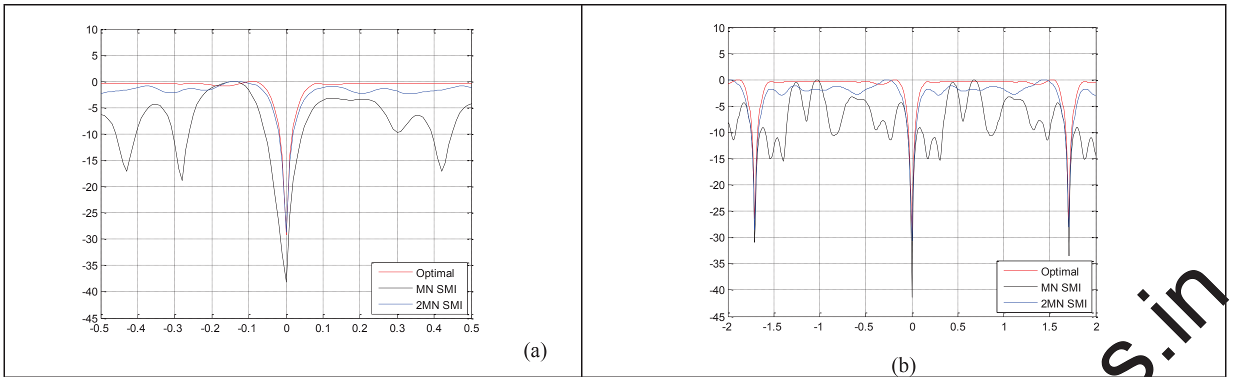

Figure 3. Improvement Factor for the DFP with constant PRF for known and unknown R, N = 8, M = 10, $d/\lambda = 0.5$, r=20: (a) $PRF = 4V_R/\lambda$, (b) $PRF = 2V_R/\lambda$

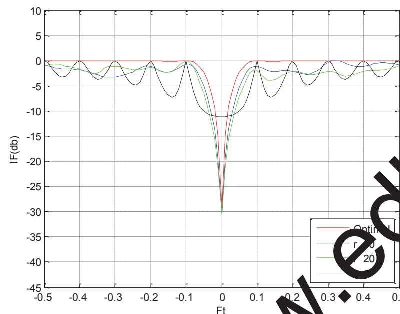

Figure 4. Improvement Factor for the PC-DFP with different values of r and with $PRF = 4V_R/\lambda$

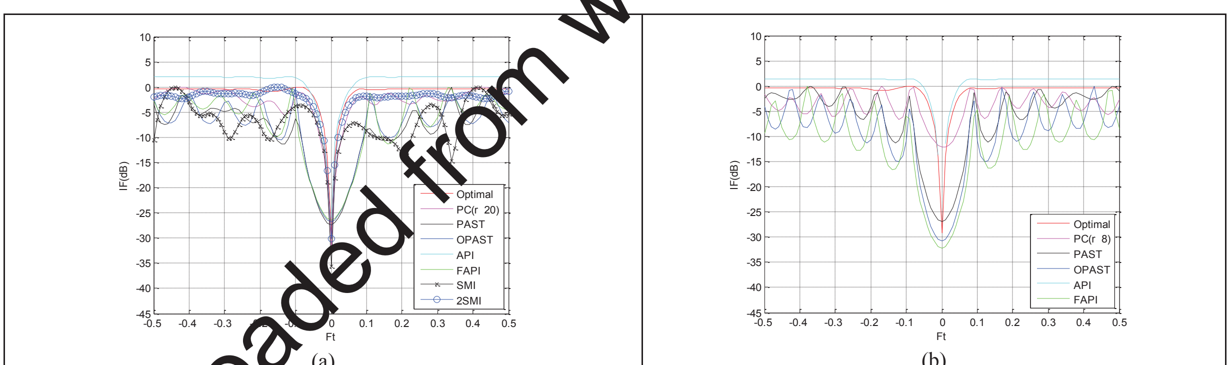

Figure 5. Improvement Factor for the iterative algorithms and PC-DFP with $PRF = 4V_R/\lambda$: (a) $r = 20$; (b) $r = 8$

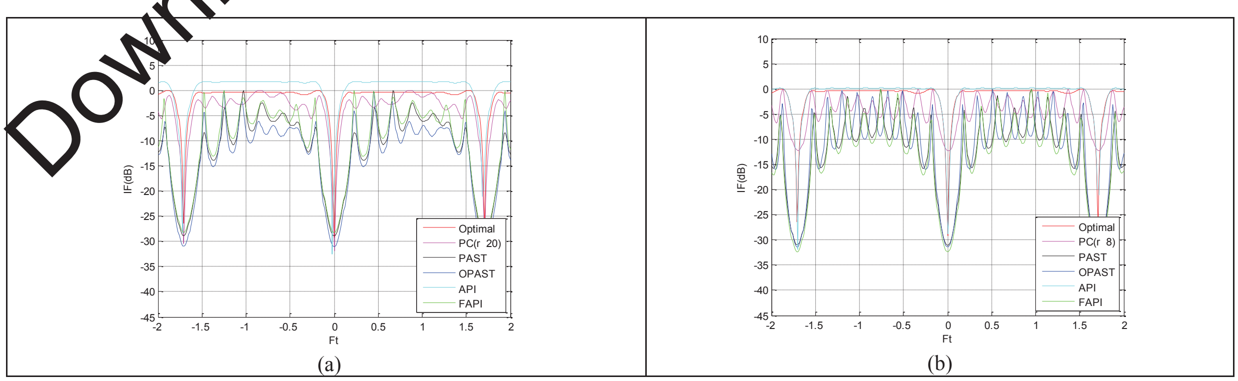

Figure 6. Improvement factor for the iterative algorithms and PC-DFP with $PRF = 2V_R/\lambda$: (a) $r = 20$; (b) $r = 8$

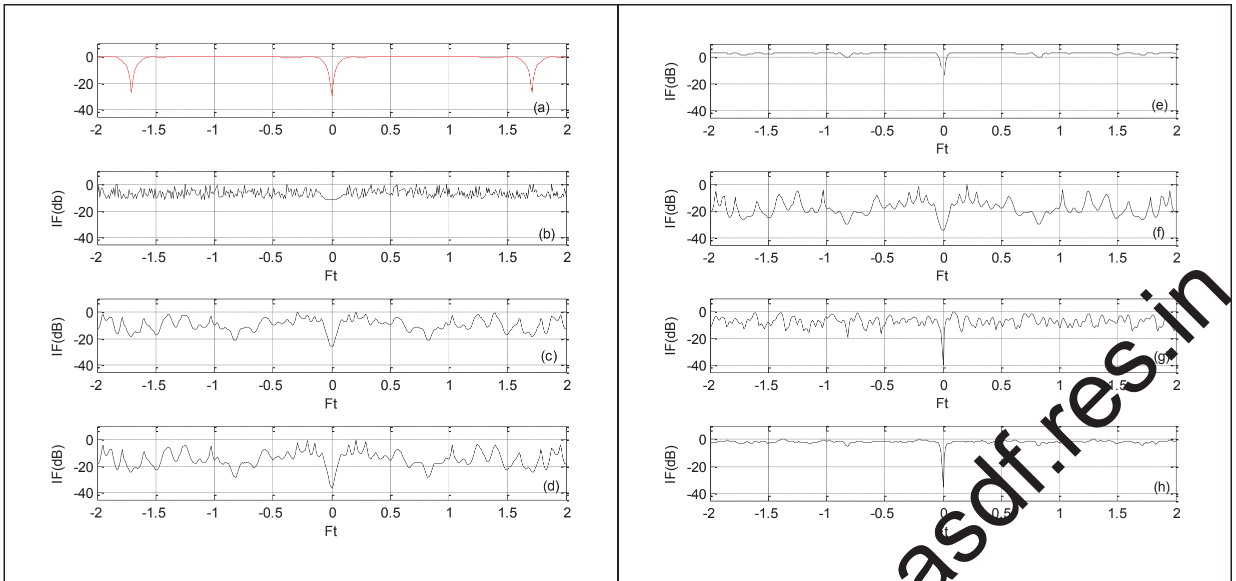

Figure 7: Improvement Factor with **pseudorandom** change of *PRF,* **(r = 8)** for PC, PAST, OPAST and FAPI algorithms: (a) Optimal detector with constant PRF (PRF=2.V_R/λ) ; (b) DFP- PC; (c) PAST; (d) OPAST; (e) API; (f) FAPI; (g) *NM* SMI; (h) *2NM* SMI

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