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Some More Results on b-δ-Closed Sets

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Abstract: *The purpose of this paper is to introduce and investigate the notions of b -δ-closed sets. We investigate some of the fundamental properties of this class of functions.*

Keywords: *b-open set, δ-open set, b-θ –open set and b-δ-open set.*

1. INTRODUCTION AND PRELIMINARIES

Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology and real analysis concerns the various modified forms of continuity, separation axioms etc., by utilizing generalized open sets. In 1963, Levine [3] also introduced the concept of semi open sets in topological space. Since then numerous applications have been found in studying different types of continuity like maps and separation of axioms. The concept of δ -interior, δ -closure, θ -interior and θ -closure operators were first introduced by Velico [8] in 1968, for the purpose of studying the important class of H-closed spaces. These operators have since been studied intensively by many authors. In 1965, Njastad [5] introduced the concept of α-open sets. Latter in 1982, Mashhour.et.al., [4] introduced the concept of pre open sets. Latter in 1996, Andrijevic [1] introduced a class of generalized open sets in a topological space, so called b-open sets. The class of b-open sets is contained in the class of semi preopen sets and contains all semi-open sets and pre-open sets.

In 1996, by using δ-closure operator Dontchev et.al., [2] introduced and studied the concept of δ-g closed sets which is a slightly stronger form of g-closedness, properly placed between δ-closedness and g-closedness and introduced the notion of $T_{\{3/4\}}$ spaces as the spaces where every δ-closed set is δ-closed. The notions of b-δ - open Set was introduced and studied by Padmanaban [6] in 2013. In this paper we investigate some more results and fundamental properties of b-δ-closed sets.

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X, τ). We denote closure and interior of A by cl (A) and int(A), respectively.

A subset A of a topological space (X, τ) is said to be

I. α -open [5] if $A \subseteq int(cl(int(A)))$,
II. Semi-open [3] if $A \subseteq cl(int(A))$,
III. Pre-open [4] or nearly open [33] if $A \subseteq int(cl(A))$,
IV. b -open [1] if $A \subseteq cl(int(A)) \cup int(cl(A))$.

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The complement of above mentioned open sets are their respective closed sets. Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a δ -cluster [8] point of A, if $int(cl(U)) \cap A = \varphi$ for every open set U of X containing x. The set of all δ - cluster points of A is called the δ -closure of A and is denoted by δ- cl(A). Alternatevely the δ -closure of A is the set of all x in X such that the interior of every closed neighborhood of x intersects A non trivially. If A = δ - cl(A), then A is called δ-closed.The complement of a δ -closed set is called δ -open set. The δ -interior of a subset A of X is defined as the union of all regular open sets of (X, τ) contained in A and is denoted by δ - int(A). The family of all δ -open subsets of X is denoted by $\delta O(X)$ and the family of all δ -closed subsets of X is denoted by $\delta C(X)$.

Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a θ -cluster [8] point of A, if $cl(U) \cap A = \varphi$ for every open set U of X containing x. The set of all θ -cluster points of A is called the θ -closure of A and is denoted by θ - cl(A)). If A = θ - cl(A), then A is called θ -closed. The complement of a θ -closed set is called θ -open set [95]. θ -interior of a subset A of X is defined as the union of all θ -open sets contained in A and is denoted by θ - int(A). The family of all θ -open subsets of X is denoted by $\theta O(X)$ and the family of all θ -closed subsets of X is denoted by $\theta C(X)$.

Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a b - θ- cluster [7] point of A, if b - $cl(U) \cap A = \varphi$ for every b -open set U of X containing x. The set of all b - θ -cluster points of A is called the b -θ -closure of A and is denoted by b - θ - cl(A). If A = b -θ - cl(A) then A is called b - θ -closed. The complement of a b -θ -closed set is called b - θ -open set. The family of all b - θ -open subsets of X is denoted by $B\theta O(X)$ and the family of all b - θ -closed subsets of X is denoted by $B\theta C(X)$.

On b-δ-Closed Sets

Definition 3.1. Let A be a subset of a topological space (X, τ). A point x of X is called a b-δ-cluster [6] point of A if $int(b\text{-}cl(U)) \cap A \neq \varphi$ for every b-open set U of X containing x. The set of all b-δ-cluster point of A is called b-δ-closure of A and is denoted by b-δ-cl(A). A subset A of a topological space (X, τ) is said to be b-δ-closed, iff A = b-δ-cl(A). The complement of a b-δ-closed set is said to be b-δ-open set.

The b-δ-interior of a subset A of (X, τ) is defined as the union of all b-δ-open sets of (X, τ) contained in A and is denoted by b-δ-int(A). Alternatively, a point x in X is called b-δ-interior point of A, if there exists a b-open sets containing x such that $int(b\text{-}cl(U)) \subseteq A$. The set of all b-$\delta$-interior points of A is called b-δ-interior of A. The family of all b-δ-open sets of the space (X, τ) is denoted by $B\delta\ O(X)$ and the family of all b-δ-closed sets of the space (X, τ) is denoted by $B\delta\ C(X)$.

Theorem 3.2. For a topological space (X, τ) the following hold:

1. Every b-δ-open is b-θ-open set,
2. Every b-θ-open is b-open set.

Proof. Follows from the definitions b-δ-open, b-θ-open and b-open sets.

The converse of above theorem need not to be true as shown in the following examples.

Example 3.3. Let X={a,b,c,d} and τ ={φ, {a}, {b}, {a,b},{b,c}, {a,b,c}, X}. Then we have BO(X,τ)={φ, {a},{b},{a,b},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d},X}, B θ O(X)={φ, {a}, {a,d},{b,c}, {a,b,c}, {b,c,d}, X} and B δ O(X)={φ, {a,d},{b,c,d}, X}.Here A={a,b,c} is b-θ-open but not b-δ-open set.

Example 3.4. Let X={a,b,c,d} and τ ={φ,{a},{b},{a,b},{a,d},{a,b,d}, X}. Then we have BO(X,τ)={φ, {a},{b},{a,b},{a,c},{a,d},{b,c},{a,b,c},{a,b,d},{a,c,d},X } and B θ O(X)={φ, {b}, {a,d},{b,c}, {a,b,d}, {a,c,d}, X}.Here A={a,b,c} is b-open but not b-θ-open set.

Theorem 3.5. For a topological space (X, τ) , Every b-δ-open is b-open set.

Proof. Follows from Theorem 3.2.

The converse of above theorem need not to be true as shown in the following examples.

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Example 3.6. Let X={a,b,c,d} and τ ={φ,{a},{b},{a,b}, X} . Then we have BO(X,τ)={φ, {a},{b},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d},{a,c,d},X } and Bδ O(X) ={φ, {a,c,d},{b,c,d}, X}.Here A={a,b,d} is b-open but not b-δ-open set.

Remark 3.7. In a topological space (X, τ), δ-open and b-δ-open are independent as shown in the following example.

Example 3.8. Let X={a, b, c}and τ ={φ, {a}, {b}, {a,b}, X}.Then we have BO(X)= {φ, {a}, {b}, {a, b}, {b,c},{a,c}, X}, δO(X)={φ, {a}, {b}, {a, b}, X}and Bδ O(X)={ φ , {b,c}, {a,c}, X}.Here A={a,b} is δ-open but not b-δ-open set and B={a,c} is b-δ-open but not δ-open set.

Remark 3.9. In a topological space (X, τ) pre-open and b-δ-open are independent as shown in the following example.

Example 3.10. Let X= {a,b,c,d} and τ ={φ, {a}, {b}, {a,b}, X}. Then we have BO(X,τ)={φ, {a}, {b}, {a,b}, {a,c}, {a,d}, {b,c},{b,d}, {a,b,c}, {a,b,d}, {b,c,d}, {a,c,d}, X},PO(X)= {φ, {a}, {b}, {a,b}, {a,b,c}, {a,b,d}, X}and Bδ O(X)={φ, {a,c,d}, {b,c,d}, X}. Here A={a,b,c} is pre-open but not b-δ-open set and B={a,c,d} is b-δ-open but not pre-open set.

Remark 3.11. In a topological space (X, τ), α-open and b-δ-open are independent as shown in the following example.

Example 3.12. Let X= {a,b,c} and τ ={φ, {a}, {b}, {a,b}, X}. Then A= {a,b} is α-open but not b-δ-open and B={b,c }is b-δ-open but not α-open set.

Remark 3.13. In a topological space (X, τ) semi-open and b-δ-open are independent, since every α-open is semi-open set.

Example 3.14 Let X= {a,b,c} and τ ={φ, {a}, {b}, {a,b}, {b,c}, X}. Then A={a,b} is semi-open but not b-δ-open set.

Example 3.15 Let X={a,b,c,d} and τ={φ, {a}, {b}, {a,b}, {a,d}, {a,b,d}, X}.Then B={a,c,d} is b-δ-open but not semi-open set.

Remark 3.16. In a topological space (X, τ), open and b-δ-open are independent as shown in the following example.

Example 3.17. Let X={a, b, c} and τ ={φ, {a}, {b}, {a,b}, X}.Then we have BO(X)= {φ, {a}, {b}, {a, b}, {b,c},{a,c}, X},δO(X)={φ, {a}, {b}, {a, b}, X}and Bδ O(X)={φ, {b,c}, {a,c}, X}.Here A={a,b} is open but not b-δ-open set and B={a,c} is b-δ-open but not open set.

Remark 3.18. The union of two b-δ-closed sets is not necessarily b-δ-closed.

Example 3.19. Let X = {a, b, c, d} and τ = {φ, {a}, {b}, {a, b}, X}.Then we have BO(X,τ)={φ, {a}, {b}, {a,b}, {a,c}, {a,d}, {b,c},{b,d}, {a,b,c}, {a,b,d}, {b,c,d}, {a,c,d}, X }and BδC(X)={φ, {a}, {b}, X }. A = {a} and B ={b} are b-δ-closed sets, but A ∪ B is not a b-δ-closed set.

Lemma 3.20. Let A be a subset of a topological space (X, τ) Then b-δ-cl(A) ⊆ bθ-cl(A).

Proof. Since int(b-cl(U)) ⊆ b-cl(U), the proof follows from definitions of b-δ-closure and b-θ-closure.

Theorem 3.21. Let A and B be any subsets of a topological space (X, τ). If A ⊆ B, then

1. 1.b-δ-cl(A) ⊆ b-δ-cl(B),
2. b-δ-cl(b-δ-cl(A)) ⊆ b-δ-cl(A).

Proof. 1. Given A ⊆ B. If U ∈ BO(X,x), then b-cl(U) ∩ A ⊆ b-cl(U) ∩ B. Hence we have, int(b-cl(U)) ∩ A ⊆ int(b-cl(U)) ∩ B. Thus b-δ-cl(A) ⊆ b-δ-cl(B)

2. Obvious.

Remark 3.22. Let A and A_α (α ∈ Λ) be any subset of a topological space (X,τ) Then the following properties hold:

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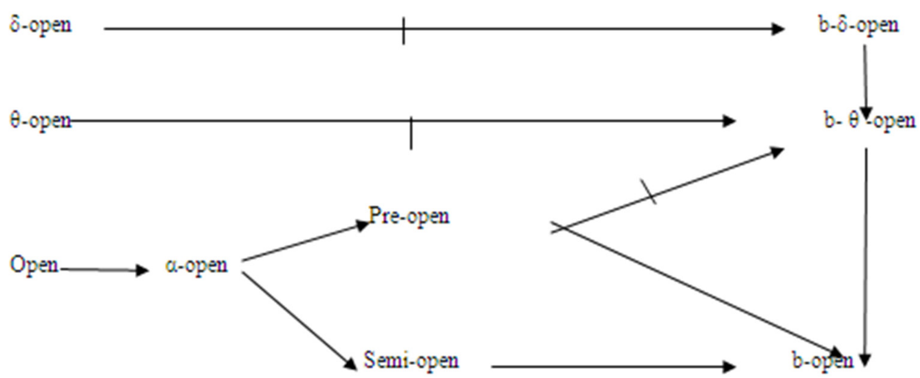
1. b-δ-cl(A) is b-δ-closed,
2. If A_α is b-δ-open in X for each $\alpha \in \Lambda$,then $\cup_{\alpha \in \Lambda} A_\alpha$ is b-δ-open in (X,τ).

Lemma 3.23. Let (X, τ) be a topological space and let A ⊆ X. Then

1. b-δ-cl(X-A) = X-(b-δ-int(A)),
2. b-δ-int(X-A) = X-(b-δ-cl(A)).

Proof. Obvious.

From the above discussion we have the following diagram.



Theorem 3.24. A subset U of a topological space (X,τ) is b-δ-open in (X, τ) if and only if for each x in U, there exists a W ∈ BO(X) with x ∈ W such that int(b-cl(W)) ⊆ U.

Proof. Suppose that U is b-δ-open in (X,τ). Then X-U is b-δ-closed. Let x ∈ U. Then x ∉ b-δ-cl(X-U) and so there exists W ∈ BO(X,x) such that int(b-cl(W)) ∩ (X-U)= φ which implies int(b-cl(W)) ⊆ X-(X-U)=U. Thus int(b-cl(W)) ⊆ U. Conversely, assume that U is not a b-δ-open. Then X - U is not b-δ-closed, and so there exists x ∈ b-δ-cl(X-U) such that x ∉ (X - U). Since x ∈ U, by hypothesis, there exists W ∈ BO(X,x) such that int(b-cl(W)) ⊆ U. Thus int(b-cl(W)) ∩ (X - U) = φ. This is a contradiction since x ∈ b-δ-cl(X - U).

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