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On b-δ- Irresolute Functions

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Abstract: *The purpose of this paper is to introduce and investigate the notions of b-δ-Irresolute functions in topological spaces. We investigate some of the fundamental properties of this class of functions.*

Keywords: *b-open set, δ-open set, b-δ-open set and b-δ-Irresolute functions.*

1. INTRODUCTION

The notions of δ -open sets, δ -closed set where introduced by Velicko [14] for the purpose of studying the important class of H-closed spaces. 1996, Andrijević [2] introduced a new class of generalized open sets called b-open sets in a topological space. This class is a subset of the class of β-open sets [1]. Also the class of b-open sets is a superset of the class of semi- open sets [8] and the class of preopen sets [8]. The purpose of this paper is to introduce and investigate the notions of b-δ-Irresolute functions. We investigate some of the fundamental properties of this class of functions. We recall some basic definitions and known results

2. Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X, τ). We denote closure and interior of A by cl (A) and int(A), respectively.

A subset A of a space X is said to be b-open [2] if $A \subseteq cl(int(A)) \cup int(cl(A))$.The complement of a b-open set is said to be b-closed. The intersection of all b-closed sets containing $A \subseteq X$ is called the b-closure of A and shall be denoted by bcl (A). The union of all b-open sets of X contained in A is called the b-interior of A and is denoted by bint (A). A subset A is said to be b-regular if it is b-open and b-closed. The family of all b-open (resp. b-closed, b-regular) subsets of a space X is denoted by BO(X) (resp. BC (X), BR(X)) and the collection of all b-open subsets of X containing a fixed point x is denoted by BO(X, x). The sets BC (X, x) and BR(X, x) are defined analogously.

A point $x \in X$ is called a δ -cluster [14] point of A if $int(cl(U)) \cap A \neq \varphi$ for every open set U of X containing x. The set of all δ -cluster points of A is called the δ - closure of A and is denoted by δ -cl (A)). A subset A is said to be δ -closed if δ –cl (A) = A. The complement of a δ -closed set is said to be δ-open. The δ -interior of A is defined by the union of all δ -open sets contained in A and is denoted by δ-int (A)).

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A point $x \in X$ is called a b-δ-cluster [10] point of A if $int(bcl(U)) \cap A \neq \varphi$ for every b-open set U of X containing x. The set of all b-δ-cluster points of A is called the b-δ-closure of A and is denoted by b-δ-cl (A)). A subset A is said to be b-δ-closed if b-δ-cl(A) = A. The complement of a b-δ-closed set is said to be b-δ-open. The b-δ-interior of A is defined by the union of all b-δ-open sets contained in A and is denoted by b-δ-int (A)). The family of all b-δ-open (resp. b-δ-closed) sets of a space X is denoted by BδO(X, τ) (resp. BδC(X, τ)).

A function f: (X, τ) →(Y, σ) is said to be

1. b-continuous [6] , if for each $x \in X$ and each open set V of Y containing f(x), there exists $U \in BO(X,x)$ such that $f(U) \subseteq V$,
2. δ-continuous function [9], if for each $x \in X$ and each open set V containing f(x), there is an open set U containing (X, τ) such that $f(int(cl(U))) \subseteq int(cl(U))$,
3. b-δ-continuous [3] (briefly b-δ-c), if for each $x \in X$ and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that $f(int (b\text{-}cl(U))) \subseteq cl(V)$.
4. Irresolute [7] if $f^{-1}(V)$ is semi-open in (X, τ) for every semi-open set V contained in (Y, σ), b-irresolute [12] if $f^{-1}(V) \in BO(X)$ for every $V \in BO(Y)$,
5. Weakly b-irresolute [13] if for each $x \in X$ and each $V \in BO(Y, f(x))$, there exists a $U \in BO(X, x)$ such that $f(U) \subseteq b\text{-}cl(V)$,
6. Strongly b-irresolute [12] if for each $x \in X$ and each $V \in BO(Y, f(x))$, there exists a $U \in BO(X, x)$ such that $f(b\text{-}cl(U) \subseteq V$.

Lemma 2.1 [11] For a topological space the following are equivalent:

1. X is b-regular,
2. For each point $x \in X$ and for each open set U of (X, τ) containing x, there exists $V \in BO(X)$ such that $x \in V \subseteq b\text{-}cl(V) \subseteq U$,
3. For each subset A of X and each closed set F such that $A \cap F = \varphi$, there exist disjoint $U, V \in BO(X)$ such that $A \cap U \notin \varphi$ and $F \subseteq V$,
4. For each closed set F of X, $F = \cap\, b\text{-}cl(V) : F \subseteq V$ and $V \in BO(X)$.

Lemma 2.2 [11] Let A and X_0 be subsets of a space X such that $A \subseteq X_0 \subseteq X$. Let $b\text{-}cl_{X0}(A)$ denote the b-closure of A with respect to the subspace X0.

1. If X_0 is α-open in X, then $b\text{-}cl_{X0}(A) \subseteq b\text{-}cl(A)$,
2. If $A \in BO(X_0)$ and $X_0 \in \alpha O(X)$, then $b\text{-}cl(A) \subseteq b\text{-}cl_{X0}(A)$.

Lemma 2.3 [5] Let A and X_0 be subsets of a topological space (X, τ).

If $A \in BO(X)$ and $X_0 \in \alpha O(X)$, then $A \cap X_0 \in BO(X_0)$,
If $A \in BO(X_0)$ and $X0 \in \alpha O(X)$, then $A \in BO(X)$.

3. b-δ- Irresolute Function

Definition 3.1 A function f: (X, τ) ⟶(Y, σ) is said to be b-δ-irresolute function if for each $x \in X$ and each open set V of (Y, σ) containing f(x), there exists a b-open set U in (X, τ) containing x such that $f (int (b\text{-}cl (U))) \subseteq b\text{-}cl(V)$.

Theorem 3.2

1. Every b-continuous function is b-δ-irresolute function,
2. Every b-δ-irresolute function is b-δ-continuous function.

Proof. Obvious.

Remark 3.3 The converse of the above theorem need not be true as shown in the following examples.

Examples 3.4 Let X = {a, b, c} and τ = {φ, {a}, {a,b}, {a,c}, X}. Define a function f: (X, τ) ⟶(Y, σ) by f(a)=b, f(b)=a and f(c)=a. We have BO(X) = {φ, a, {a, b}, {a, c}, X}. Then f is b-δ irresolute function but not b-continuous function, since for V = {a} and V = {a, c} there exists no $U \in BO(X, x)$ for x=b and x=c such that $f(U) \subseteq V$

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Examples 3.5 Let X = {a, b, c, d} and τ = {φ, {a}, {b}, {a,b}, {a,d}, {a,b,d}, X}. Define a function f: (X, τ) ⟶ (Y, σ) by f (a) = b, f (b) = c, f (c) = b and f (d) = c. Then we have BO (X) = {φ, {a}, {b}, {a, b}, {a, c}, {a,d}, {b, c},{a, b, c}, {a, b, d}, {a, c, d}, X}, Then f is b-δ-continuous function but not b-δ irresolute function, since for V ={b} there exists no U∈ BO(X, x) for x=a and x=c such that f(int(b-cl(U))) ⊆ b-cl(V).

Remark 3.6 From the above discussions we have the following diagram. None of the implications are reversible. b-continuous function ⟶ b-δ-irresolute function ⟶ b-δ- continuous function.

Theorem 3.7 For a function f: (X, τ) ⟶ (Y, σ), the following are equivalent:

1. F is b-δ-irresolute,
2. b-δ-cl(f^{-1}(B))⊆ f^{-1}(b-δ-cl(B)) for every subset b of (Y, σ),
3. f(b-δ-cl(A))⊆ b-δ-cl(f(A)) for every subset A of (X, τ).

Proof. 1.⟶ 2. Let B be any subset of (Y, σ). Suppose that x ∉ f^{-1} (b-δ-cl (B)). Then f (x) ∉ b-δ-cl (B) and there exists V∈ BO(Y, f(x)) such that int (b-cl (V)) ∩ B =φ. Since f is b-δ-irresolute, there exists U∈ BO(X, x) such that f (int (b-cl (U))) ⊆ b-cl (V). Therefore, we have f (int (b-cl (U))) ∩ B= φ and int (b-cl (U)) ∩ f^{-} (B)=φ. This shows that x ∉ b-δ-cl (f^{-1}(B)). Hence, we obtain b-δ-cl (f^{-1}(B)) ⊆ f^{-1}(b-δ-cl (B)).
2. ⟶ 3. Let A be any subset of (X, τ). Then we have b-δ-cl(A) ⊆ b-δ-cl(f^{-1}(f(A))) ⊆ f^{-1}(b-δ-cl(f(A))) and hence f(b-δ-cl(A))⊆ b-δ-cl(f(A)).
3. ⟶ 2.: Let b be a subset of (Y, σ). We have f(b-δ-cl(f^{-1}(B))) ⊆ b-δ-cl(f(f^{-1}(B))) ⊆ b-δ-cl(B) and hence b-δ-cl(f^{-1}(B))⊆ f^{-1}(b-δ-cl(B)).
2. ⟶ 1.: Let x∈ X and V∈ BO(Y, f(x)). Then we have int (b-cl (V))) ∩ (Y - (b-cl (V))) = φ and f(x) ∉ b-δ-cl (Y-(b- cl (V))). Hence, x ∉ f^{-1} (b-δ-cl(Y-(b-cl (V)))) and x ∉ (b-δ-cl (f^{-1}(Y-(b-cl (V)))). There exists U∈ BO(X, x) such that int (b-cl (U)) ∩ f^{-1}(Y-(b –cl (V))) = φ and hence f (int (b-cl (U))) ⊆ b-cl (V). This shows that f is b-δ-irresolute.

Theorem 3.8 For a function f: (X, τ) ⟶ (Y, σ), the following are equivalent:

1. F is b-δ-irresolute,
2. f^{-1}(V) ⊆ b-δ-int (f^{-1}(b-cl (V))) for every V∈ BO(Y),
3. B-δ-cl (f^{-1}(V)) ⊆ f^{-1}(b-cl (V)) for every V∈ BO(Y).

Proof. 1. ⟶ 2. Suppose that V∈ BO(Y) and x∈ f^{-1}(V). Then f(x) ∈ V and there exists U∈ BO(X,x) such that f(int(b-cl(U)))⊆ b-cl(V). Therefore, x∈ U such that int(b-cl(U)) ⊆ f^{-1} (b-cl(V)). This shows that x∈ b-δ-int(f^{-1}(b-cl(V))). This shows that f^{-1}(V) ⊆ b-δ-int(f^{-1}(b-cl(V))).
2. ⟶ 3. Suppose that V∈ BO(Y) and x ∉ f^{-1}(b-cl(V)). Then f(x) ∉ b-cl(V) and there exists U∈ BO(Y, f(x)) such that U}∩ V=φ and hence int(b-cl(U))∩ V=φ Therefore, we have f^{-1}(int(b-cl(U)))∩ f^{-1}(V)=φ Since x∈ f^{-1}(U), by(2), x∈ b-δ-int(f^{-1}(b-cl(U))). There exists W∈ BO(X,x) such that int(b-cl(W))⊆ f^{-1}(b-cl(U)). Thus, we have int(b-cl(W))∩ f^{-1}(V)=φ and hence x ∉ b-δ-cl(f^{-1}(V)). This shows that b-δ-cl(f^{-1}(V))⊆ f^{-1}(b-cl(V)).
3. ⟶ 1.: Suppose that x∈ X and V∈ BO(Y, f(x)). Then, V∩ (Y-b-cl(V))=φ and f(x) ∉ int(b-cl(Y-b-cl(V))). Therefore, x ∉ (int(b-cl(Y-b-cl(V)))) and by(3), x ∉ b-δ-cl(f^{-1}(Y-b-cl(V))). There exists U∈ BO(X,x) such that int(b-cl(U))∩ f^{-1}(Y-b-cl(V))=φ. Therefore, we obtain f(int(b-cl(U)))⊆ b-cl(V). This shows that f is b-δ- irresolute.

Theorem 3.9 Let (Y, σ) be a b- regular space. Then for a function f: (X, τ) ⟶ (Y, σ), the following are equivalent:

1. f is strongly b-irresolute,
2. f is b-irresolute,
3. f is b-δ- irresolute.

Proof. 1. ⟶ 2. obvious.
2. ⟶ 3. Suppose that x∈ X and V∈ BO(Y, f(x)). Since f is b-irresolute, f^{-1}(V) is b-open and f^{-1}(bcl(V)) is b-closed in X. Now, put U= f^{-1}(V). Then we have U∈ BO(X,x) and int(bcl(U))⊆ f^{-1}(bcl(V)). Therefore, we obtain f(int(bcl(U)))⊆ bcl(V). This shows that f is b-δ- irresolute.
3. ⟶ 1. Suppose that x∈ X and V∈ BO(Y, f(x)). Since Y is b- regular, there exists W∈ BO(Y) such that f(x) ∈ W ⊆ bcl(W)⊆ V, by Lemma 2.1 . Since f is b-δ- irresolute, there exists U∈ BO(X,x) such that f(int(bcl(U)))⊆ f(bcl(U)) ⊆ bcl(W)⊆ V. This shows that f is strongly b-irresolute.

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Theorem 3.10 Let (X, τ) be a b- regular space. Then f: (X, τ) ⟶ (Y, σ) is b-δ- irresolute if and only if it is weakly b-irresolute.

Proof. Suppose that f is weakly b-irresolute. Let x∈ X and V∈ BO(Y, f(x)). Then, there exists U∈ BO(X, x) such that f(U)⊆ b-cl(V). Since X is b- regular, there exists U_0∈ BO(X, x) such that x∈ U0 ⊆ (b-cl(U_0))⊆ U, by Lemma 2.1 . Therefore, we obtain f(b-cl(U_0)⊆ b-cl(V). Hence f(int(b-cl(U_0))⊆ b-cl(V). This shows that f is b-δ- irresolute.

Theorem 3.11 If f: (X, τ) ⟶ (Y, σ) is b-δ- irresolute and X_0 is an α -open subset of X, then the restriction $f|_{X0}$: X_0 ⟶ Y is b-δ- irresolute.

Proof. For any x∈ X_0 and any V∈ BO(Y, f(x)), there exists U∈ BO(X,x) such that f(int(b-cl(U))) ⊆ b-cl(V) since f is b-δ- irresolute. Let U_0=U∩ X_0, then by Lemma 2.2 and Lemma 2.3. U_0∈ BO(X_0,x) and $b\text{-}cl_{X0}$ (U_0)⊆ b-cl(U_0). Hence int($b\text{-}cl_{X0}$(U_0))⊆ b-cl(U_0). Therefore, we obtain ($f|_{X0}$) (int($b\text{-}cl_{X0}$(U_0)))= f(int($b\text{-}cl_{X0}$(U_0)))⊆ f(b-cl(U_0))⊆ f(b-cl(U))⊆ b-cl(V). This shows that $f|_{X0}$ is b-δ- irresolute.

Theorem 3.12 f: (X, τ) ⟶ (Y, σ) is b-δ- irresolute if for each x∈ X there exists X_0 ∈ αO (X, x) such that the restriction $f|_{X0}$: X_0 ⟶ Y is b-δ- irresolute.

Proof. Let x∈ X and V∈ BO(Y, f(x)). There exists X_0 ∈ α O (X, x) such that $f|_{X0}$: X_0⟶ Y is b-δ- irresolute. Thus, there exists U∈ BO(X_0, x) such that ($f|_{X0}$ (int($b\text{-}cl_{X0}$U))) ⊆ b-cl(V). By Lemma 2.2 and Lemma 2.3. U∈ BO(X, x) and b-cl(U)⊆ $b\text{-}cl_{X0}$(U). Hence, int(b-cl(U))⊆ $b\text{-}cl_{X0}$(U). Thus we have f(int(b-cl(U)))=($f|_{X0}$(int(b-cl(U)))⊆ ($f|_{X0}$) (int($b\text{-}cl_{X0}$(U)))⊆ b-cl(V). This shows that f is b-δ- irresolute.

Corollary 3.13 Let {$U_α$: α ∈ Λ} be an α -open cover of a topological space (X, τ). A function f: (X, τ) ⟶ (Y, σ) is b-δ- irresolute if and only if the restriction $f|_{Uα}$: $U_α$ ⟶ Y is b-δ- irresolute for each α ∈ Λ .

Proof. Follows from Theorems 3.11 and Theorems 3.12

Theorem 3.14 Let f: (X, τ) ⟶ (Y, σ), g: (Y, σ) ⟶(z,η) be functions and go f : (X, τ) ⟶ (z,η) be the composition. Then the following hold:

1. If f and g are b-δ- irresolute, then go f is b-δ-irresoloute,
2. If f is strongly b-irresolute and g is weakly b-irresolute, then go f is b-δ-irresoloute,
3. If f is weakly b-irresolute and g is b-δ-irresolute, then go f is weakly b-irresolute,
4. If f is b-δ-irresoloute and g is strongly b-irresolute, then go f is strongly b-irresolute.

Proof. Obvious.

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