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Relationship between Closed Sets in Topological Spaces

J Logeshwari[1], J Manonmani[2]

Abstract: *This paper shows the relationship between several closed sets in the topological spaces. This study is made on , , , - closed sets .*

Keywords :, , , *sets.*

INTRODUCTION

Topology as a branch of mathematics can be formally defined as "the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map), especially those properties that are invariant under a certain kind of equivalence (called homeomorphism)". Simply, Topology is the study of continuity and connectivity. The first step of generalizing closed sets was done by Levine in 1970. Syed Ali Fathima and Mariasingam was introduced regular generalized open sets. Pre g∗ -closed sets was introduced by Jafari, Benchalli, Patil and Rayagoudar. Parimelazhagan and Subramonia Pillai introduced strongly g∗ -closed sets. b∗ -closed sets and strongly b∗ -closed sets were introduced by Muthuvel and Parimelazhagan, Poongothai and Parimelazhagan respectively. β^-generalized closed sets and open sets was introduced by Kannan and Nagaveni. Pushpalatha and Nithyakala introduced sc∗g -closed sets in Topological spaces.

Preliminaries

Definition 1.1. A subset A of a space X is called a #regular generalized closed (briefly #rg-closed) set if cl(A)⊆U whenever A⊆U and U is rw-open.

Definition 1.2. A subset A of a topological space X is called a pre g∗ -closed (briefly pg∗ -closed) set if pcl(A)⊆U whenever A⊆U and U is wα-open set in X.

Definition 1.3. Let X be a topological space and A be its subset, then A is a strongly g∗ -closed set(briefly sg∗ -closed) if cl(int(A)) ⊆ U whenever A ⊆ U and U is g-open.

Definition 1.4. A subset A of a topological space X is called a b∗ -closed set if int(cl(A))⊆U, whenever A⊆U and U is b-open.

Definition 1.5. A subset A of a topological space X is called a β^g-closed set (β^-generalized closed set) if cl(int(cl(A))) ⊆ U whenever A ⊆ U and U is openin X.

Definition 1.6. A subset A of a topological space X is called a strongly b∗ -closed (briefly sb∗ -closed) set if cl(int(A))⊆U whenever

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A⊆U and U is b-open in X.

Definition 1.7. A subset A of X is called a sc*g-closed set if scl(A) ⊆ Uwhenever, A⊆U and U is a C-set in X. The following example shows that sg^*-closed and b^*-closed sets are independent.

Example 7.1. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$. Then the subset $A = \{a\}$ is b^*-closed but not sg^*-closed in X. For the topology $\tau_2 = \{\phi,\{b,c\},X\}$ the subset $B = \{b\}$ is sg^*-closed but not b^*-closed in X. The concept of $sg^*$and $\hat{\beta}g$-closed sets are independent as seen in the next example.

Example 7.2. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{b,c\},X\}$. Then the subset $\{c\}$ is sg^*-closed but not $\hat{\beta}g$-closed in X. For the topology $\tau_2 = \{\phi,\{a\},X\}$, the subset $B = \{a,b\}$ is $\hat{\beta}g$-closed but not sg^*-closed in X.

In the next example we see that the concept of sg^*-closed and sc^*g-closed sets are independent.

Example 7.3. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{b,c\},X\}$. Then the subset $A = \{a\}$ is sg^*-closed but not sc^*g-closed in X. For the topology $\tau_2 = \{\phi,\{a\},X\}$, the subset $B = \{a,b\}$ is sc^*g-closed but not sg^*-closed in X.

The following example shows that b^*-closed and sb^*-closed sets are independent.

Example 7.4. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$. Then the subset $A = \{a\}$ is b^*-closed but not sb^*-closed in X. For the topology $\tau_2 = \{\phi,\{b,c\},X\}$ the subset $B = \{b\}$ is sg^*-closed but not b^*-closed in X.

The concept of b^*-closed and $\hat{\beta}g$-closed sets are independent as seen in the next example.

Example 7.5. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{b,c\},X\}$. Then the subset $A = \{a,b\}$ is $\hat{\beta}g$-closed but not b^*-closed in X. For the topology $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},X\}$, the subset $B = \{a\}$ is b^*-closed but not $\hat{\beta}g$ -closed in X.

In the next example we see that the concept of sb^*-closed and $\hat{\beta}g$-closed sets are independent.

Example 7.6. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a,c\},X\}$. Then the subset $A = \{b\}$ is sb^*-closed but not $\hat{\beta}g$ -closed in X. For the topology $\tau_2 = \{\phi,\{a\},\{a,c\},X\}$, the subset $B = \{a,b\}$ is $\hat{\beta}g$ -closed but not sb^*-closed in X.

The following example shows that $\hat{\beta}g$ -closed and pg^*-closed sets are independent.

Example 7.7. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a,b\},X\}$. Then the subset $A = \{a\}$ is pg^*-closed but not $\hat{\beta}g$ -closed in X. For the topology $\tau_2 = \{\phi,\{a\},\{a,b\},X\}$ the subset $B = \{a,c\}$ is $\hat{\beta}g$ -closed but not pg^*-closed in X.

The concept of $\hat{\beta}g$ -closed and sc^*g -closed sets are independent as seen in the following example.

Example 7.8. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a,b\},X\}$. Then the subset $A = \{b,c\}$ is $\hat{\beta}g$-closed but not sc^*g-closed in X. For the topology $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},X\}$, the subset $B = \{b\}$ is sc^*g -closed but not $\hat{\beta}g$ -closed in X.

In the next example we see that the concept of pg^*-closed and sc^*g -closed sets are independent.

Example 7.9. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a\},\{b,c\},X\}$. Then the subset $A = \{b\}$ is pg^*-closed but not sc^*g -closed in X. For the topology $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},X\}$, the subset $B = \{a\}$ is sc^*g -closed but not pg^*-closed in X.

Finally, in the next example we see that pg^*-closed and sb^*-closed sets does not imply $\#rg$-closed.

Example 7.10. Let $X = \{a,b,c\}$ with the topology $\tau_1 = \{\phi,\{a\},\{b,c\},X\}$. Then the subset $\{a,b\}$ is pg^*-closed and sb^* -closed sets but not $\#rg$-closed in X.

We have the following diagram for our conclusion.

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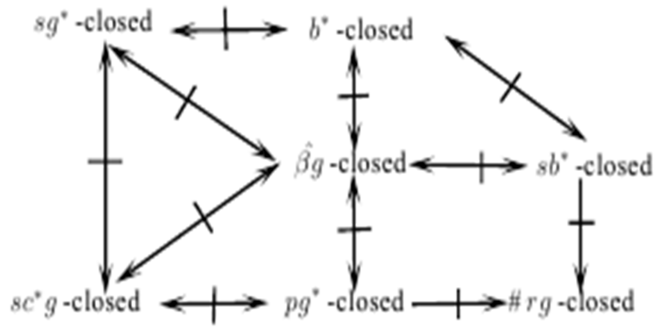

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