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A Review on Pairwise T_S - Spaces

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Abstract: The purpose of this paper is to review on pairwise TS-spaces and some of their properties.

Keywords: pairwise $T_{1/2}$ – space, pairwise T_S- space, τ1τ2 - αg closed set.

1. INTRODUCTION AND PRELIMINARIES

A triple (X, τ_1, τ_2) where X is a non empty set and $\tau_1$and τ_2 are topologies on X is called a bitopological space and Kelly [17] initiated the study of such spaces. K.Chandrasekhara Rao and K.Kannan [3, 4, 16, 6] introduced the concepts of s^*g-closed sets and $s*g$-locally closed sets in bitopological spaces. Using the concept of $\tau_1\tau_2$-s^*g-closed sets, K.Chandrasekhara Rao and D.Narasimhan [5] introduced the concept of pairwise T_s-spaces in bitopological spaces. Ideals in topological spaces have been considered since 1930. In 1990, Jankovic and Hamlett [15], once again initiated the application of topological ideals and generalized the most fundamental properties in topological spaces. An ideal I on a topological space (X ,τ) is a collection of subsets of X which satisfies the following properties: (i) $A \in I$ and $B \subseteq A$ implies $B \in I$,(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. (X, τ, I) represents the topological space with an ideal I. Let $P(X)$ be the set of all subsets of X, a set operator $(\ )^*$: $P(X) \rightarrow P(X)$, called the local function [20] of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X \ / \ U \cap A \notin I$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(I, \tau)$ in case there is no confusion. X^* is often a proper subset of X. For every ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I, \tau) = \{U \setminus J$: $U \in \tau$ and $J \in I\}$. It is known in [15] that $\beta(I, \tau)$ is not always a topology on X. A subset A of an ideal space (X, τ, I) is called τ^*-closed [15] or simply *-closed (resp. *-dense in itself) if $A^* \subseteq A$ (resp. $A \subseteq A^*$). A Kuratowski closure operator $cl^*(\ )$ for a topology $\tau^*(I, \tau)$, called the *-topology, is defined by $cl^*(A) = A \cup A^*(\tau, I)$ [31]. M.Khan and M.Hamza [19] introduced the concept of I_{s*g}-closed sets in ideal topological spaces.

Definition: 1.1 A bitopological space $(X, \tau_1, \tau 2)$ is called a

1. pairwise $T_{1/2}$-space [11] if every τ_1-g closed set is τ_2-closed and every τ_2-gclosed set is τ_1-closed,
2. pairwise $T*_{1/2}$-space [30] if every $\tau_1\tau_2$-$g*$closed set is τ_2-closed and every $\tau_2\tau_1$ –$g*$closed set is τ_1-closed,
3. pairwise T_b-space [9] if every $\tau_1\tau_2$ -gs closed set is τ_2-closed and every $\tau_2\tau_1$ - gs closed set is τ_1-closed,
4. Pairwise $T*_p$-space [32] if every $\tau_1\tau_2$ -$g*p$ closed set is τ_2 –closed.

Definition: **1.2**: A bitopological space (X, τ_1, τ_2) is called a pairwise T_S-space if every $\tau_1\tau_2$ - s^*g closed set is τ_2-closed in X and every $\tau_2\tau_1$-s^*g *closed* set is τ1-closed in X.

Example 1.3: Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, c\}, X\}$.Then $\{X, \tau_1, \tau_2\}$ is a pairwise T_S-space.

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Proposition 1.4: Let (X, τ_1, τ_2) be a $\tau1\tau2$- T_S space.

a) If Y is a τ_2-closed subspace of X, then $(Y, \tau_1/Y, \tau_2/Y)$ is a $\tau_1\tau_2$ - T_S space and
b) If Y is a τ_1-closed subspace of X, then $(Y, \tau_1/Y, \tau_2/Y)$ is a $\tau_2\tau_1$ - T_S space.

Proof. Let X be a pairwise T_S- space and Y be a τ_2 - closed subspace of X. Let A be $\tau_1\tau_2$ - s^*g *closed* in Y. Let $A \subseteq U$ and U is τ_1-semi open in Y.

Then, τ_2 - $cl_Y(A) \subseteq U$. Since U is τ_1- semi open in Y, we have $U = G \cap Y$ where G is τ_1- semi open in X. Therefore $A \subseteq G$ and G is τ_1-semi open in X. Since A is $\tau_1\tau_2$- s^*g closed in Y, we have $A = H \cap Y$ where H is $\tau_1\tau_2$- s^*g closed in X. But X is a pairwise T_S- space.
$\Rightarrow$ H is τ_2- closed in X.
$\Rightarrow$ $H \cap Y$ is τ_2- closed in X.
$\Rightarrow$ A is τ_2- closed in X.
$\Rightarrow$ $A \cap Y$ is τ_2- closed in Y.
$\Rightarrow$ A is τ_2-closed in Y.
(b) As we proved in (a)

Theorem 1.5: Let I be a index set. Let $\{X_i, i \in I\}$ be pairwise T_S-spaces. Then their product $X = \Pi X_i$ is a pairwise TS-space.

Proof. Let $A = p_j(A) \times \Pi X_i$, $i \neq j$ be $\tau_1\tau_2$- s^*g closed in $X = \Pi X_i$ where $p_j : \Pi X_i \rightarrow X_j$ be the j[th] projection map which is a surjection. Then $\tau2$- cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- semi open in X. Since U is τ_1- semi open in $X = \Pi X_i$, $U = \Pi X_i \times U_j$, $j \neq i$, where U_j is τ_1- semi open in X_j. Since p_j: $\Pi X_i \rightarrow X_j$, $i \neq j$, be the j^{th} projection map, we have $p_j(U) = U_j$. Also $A \subseteq U$. Hence $p_j(A) \subseteq p_j(U) = U_j$. Since A is $\tau_1\tau_2$- s^*g closed in X, $p_j(A)$ is $\tau_1\tau_2$- s^*g closed in X_j. Since X_j is a pairwise T_S- space, we have $A_j = p_j(A)$ is τ_2- closed in X_j. Hence $A_j = \tau_2$ -$cl_{Xj}(A_j)$. Therefore $A_j \times \Pi X_i = \tau_2$-$cl_{Xj}(A_j) \times \Pi X_i = \tau_2$-cl$(A_j) \times \Pi X_i = \tau_2$-cl$[(A_j) \times \Pi X_i]$. Hence A is τ_2 -closed in X. Therefore every $\tau_1\tau_2$- s^*g closed set is τ_2- closed. Similarly, we can prove every $\tau_2\tau_1$- s^*g closed set is τ_1- closed. Hence X is a pairwise T_S- space.

Lemma 1.6: The inverse image of a $\tau_1\tau_2$- s^*g closed set under a pairwise continuous bijection map $f : X \rightarrow Y$ is $\tau_1\tau_2$- s^*g closed, where Y is another bitopological space.

Proof. Let $f :(X, \tau1, \tau2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise continuous bijection. Let A be $\sigma_1\sigma_2$ - s^*g closed in Y. We shall show that $f^{-1}(A)$ is $\tau_1\tau_2$ - s^*g closed in X. Let $f^{-1}(A) \subseteq U$, where U is τ_1-semi open in X. Then $A \subseteq f(U)$ and $f(U)$ is σ_1- semi open in Y. Since A is $\sigma_1\sigma_2$ - s^*g closed in Y, we have σ_2 - cl$(A) \subseteq f(U)$. Therefore τ_2 –cl $[f^{-1}(A)] \subseteq f^{-1}[\sigma_2$- cl $(A)] \subseteq f^{-1}[f(U)] = U$ {since f is pairwise continuous and bijection}.$\Rightarrow \tau_2$- cl$[f^{-1}(A)] \subseteq U$. Then $f^{-1}(A)$ is $\tau_1\tau_2$- s^*g closed in X.

Theorem 1.7: The image of a pairwise T_S-space under a pairwise continuous bijection map f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a pairwise T_S-space, where Y is another bitopological space.

Proof. Let f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pairwise continuous bijection map. Since f is onto, we have $Y = f(X)$. Let A be $\sigma_1\sigma_2$ - s^*g closed in Y. We shall show that A is σ_2 - closed in Y. By Lemma 4.5, we have $f^{-1}(A)$ is $\tau_1\tau_2$ - s^*g closed in X. But, X is a pairwise T_S-space. Hence $f^{-1}(A)$ is τ_2 - closed in X.

$\Rightarrow F\text{-}1(A) = \tau_2$-cl $[f^{-1}(A)]$. This implies $A = f[\tau2$ - cl $[f^{-1}(A)] \supseteq \sigma_2 -$ cl (A).

Hence $\sigma2 -$ cl $(A) \subseteq A$. Obviously $A \subseteq \sigma_2 -$ cl (A).

Therefore, $\sigma_2 -$ cl $(A) = A$. Now, σ_2 - $cl_Y(A) = \sigma_2$- cl $(A) \cap Y = A \cap Y = A$. Therefore, A is σ_2 - closed in Y. Similarly we can prove every $\sigma_2\sigma_1$ - s^*g closed set is σ_1 - closed in Y. Hence Y is pairwise T_S- space.

Theorem 1.8: In a pairwise T_S- space,

a) the intersection of two $\tau_1\tau_2$ - s^*g closed sets is $\tau_1\tau_2$ - s^*g closed;
b) The union of two $\tau1\tau2$ - s^*g *open* sets is $\tau1\tau2$ - s^*g open.

Proof. (a) Let A and B be two $\tau_1\tau_2$ - s^*g closed sets in (X, τ_1, τ_2). Since X is a pairwise TS-space, A and B are τ_2-closed in X. Hence $A \cap B$ is τ_2-closed in X. Consequently $A \cap B$ is $\tau1\tau2$ - s^*g closed in X.

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(b) Let A and B be two $\tau_1\tau_2$ - s^*g *open* sets in (X, τ_1, τ_2). Then A^C and B^C are τ1τ2 - s^*g closed in X. By (a), AC ∩ B^C = $(A \cup B)^C$ is τ1τ2 - s^*g closed in X. Therefore $A \cup B$ is $\tau_1\tau_2$ - s^*g open in X.

Theorem 1.9: (a) Every pairwise $T_{1/2}$ - space is a pairwise T_S- space;
(b) Every pairwise T_b - space is a pairwise T_S - space;
(c) Every pairwise ${}_\alpha T_b$ - space is a pairwise T_S- space;
(d) Every pairwise door space is a pairwise T_S- space.

Proof. (a) Suppose that X is a pairwise $T_{1/2}$ - space. Since every $\tau_1\tau_2$ - s^*g closed set is τ_2 - closed in a pairwise $T_{1/2}$ - space, X is a pairwise T_S - space.
(b) Suppose that X is a pairwise Tb - space. Let A be $\tau_1\tau_2$ - s^*g closed in X. Then A is τ1τ2 - gs closed in X. Since X is a pairwise T_b - space, A is τ_2 - closed in X. Hence X is pairwise T_S - space.
(c) Suppose that X is a pairwise αTb - space. Let A be τ1τ2 - s^*g closed in X. Then A is $\tau_1\tau_2$ - αg closed in X. Since X is a pairwise ${}_\alpha T_b$ - space, A is τ_2 - closed in X. Therefore X is a pairwise T_S- space.
(d) Let X be a pairwise door space. Then X is a pairwise $T_{1/2}$. From (a), we have X is a pairwise T_S- space.

Remark 1.10: The converse of the above theorem are not true as can be seen from the following example.

Example 1.11: In Example 4.2, (X, τ_1, τ_2) is a pairwise T_S- space but not a pairwise $T_{1/2}$ - space, pairwise T_b - space, pairwise ${}_\alpha T_b$ - space or a pairwise door space.

Theorem 1.12:

a) Every $\tau_1\tau_2$ - gs closed set in a pairwise T_b - space is $\tau_1\tau_2$ - s^*g closed;
b) Every $\tau_1\tau_2$ - sg closed set in a pairwise T_b - space is $\tau_1\tau_2$ - s^*g closed;
c) Every $\tau_1\tau_2$ - αg closed set in a pairwise ${}_\alpha T_b$ - space is $\tau_1\tau_2$ - s^*g closed.

Proof. (a) Let X be a pairwise T_b - space and A be $\tau_1\tau_2$ - gs closed in X. Then A is τ2 - closed in X. Consequently, A is $\tau_1\tau_2$ - s^*g *closed* in X.

(b) Let X be a pairwise T_b - space and A be τ1τ2 - sg closed in X. Since A is $\tau_1\tau_2$ - gs closed in X, A is $\tau_1\tau_2$ - s^*g closed in X {by (a)}.

(c) Let X be a pairwise αTb - space and A be τ1τ2 - αg closed in X. Then A is τ_2 - closed in X. Consequently, A is $\tau_1\tau_2$ - s^*g closed in X.

Corollary 1.13:

a) Every subset of a pairwise complemented T_b - space is $\tau_1\tau_2$ - s^*g closed;
b) Every subset of a pairwise complemented $T_{1/2}$ - space is $\tau_1\tau_2$ - s^*g closed;
c) Every subset of a pairwise complemented ${}_\alpha T_b$ - space is $\tau_1\tau_2$ - s^*g closed.

Proof. (a) Since X is a pairwise complemented, every subset of X is $\tau_1\tau_2$ - gs closed in X. Since X is a pairwise T_b - space, every subset of X is $\tau_1\tau_2$ - s^*g closed in X {by Theorem 4.11(a)}.

(b) Since X is a pairwise complemented, every subset of X is $\tau_1\tau_2$ - g closed in X. Since X is a pairwise $T_{1/2}$ - space, every subset of X is $\tau_1\tau_2$ - s^*g closed in X.

(c) Since X is a pairwise complemented, every subset of X is $\tau_1\tau_2$ - αg closed. Since X is a ${}_\alpha T_b$-space, every subset of X is $\tau_1\tau_2$ - s^*g closed in X by Theorem 4.11(c)}.

Theorem 1.14: If (X, τ_1, τ_2) is both pairwise T^*_p - space and pairwise ${}^*T_p$ - space then X is a pairwise T_S - space.

Proof. Let A be $\tau_1\tau_2$ - s^*g closed in X. Then A is $\tau_1\tau_2$ - gp closed in X. Since X is a pairwise ${}^*T_p$ -space, A is $\tau_1\tau_2$-g^*p closed in X. Therefore X is a pairwise T^*_p- space. Hence A is τ_2 - closed in X. Consequently, X is a pairwise T_S- space.

References

1. Arya.S.P and Nour.T, Characterizations of s-normal spaces, Indian J. Pure. Appl. Math., 21 (8) (1990), 717-719.

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2. Chandrasekhara Rao.K and Joseph.K, Semi star generalized closed sets, Bulletin of Pure and Applied Sciences 19E(No 2) 2000, 281-290.
3. Chandrasekhara Rao.K and K.Kannan, Semi star generalized closed sets and semi star generalized open sets in bitopological spaces, Varahimir Journal of Mathematics 5(2) (2005), 473-485.
4. Chandrasekhara Rao.K, Kannan.K. and Narasimhan.D., Characterizations of $\tau_1\tau_2$-$s*g$- closed sets, Acta Ciencia Indica, XXXIII M, (3) (2007), 807-810.
5. Chandrasekhara Rao.K and Narasimhan.D, Pairwise *Ts*-Spaces, Thai Journal of Mathematics 6(1) (2008), 1-8.
6. Chandrasekhara Rao.K and Kannan.K, s*g-Locally Closed Sets in Bitopological spaces Int.J.Contemp.Math.Sciences, 4(12) (2009), 597-607.
7. Crossley.S.G and Hilderbrand.S.K, semi-closure, Texas J. Sci., 22(1971), 99-112.
8. Dontchev.J, Ganster.M and Noiri.T, Unified approach of generalized closed sets via topological ideals, Math.Japan, 49(1999), 395-401.
9. El-Tantawy.O.A. and Abu-Donia.H.M, Generalized Separation axioms in Bitopological spaces, The Arabian Journal for Science and Engg.30 (1A) 2005, 117-129.
10. Fathi H.Khedr and Hanan S.Al-saadi, On pairwise semi generalized closed sets JKAU: sci, 21(2) 2009, 269-295.
11. Fukutake.T, On generalized closed sets in bitopological spaces, Bull.Fukuoka.Univ.Ed. Part III, 35(1986), 19-28.
12. Fukutake.T, Semi open sets on bitopological spaces, Bull.Fukuoka Uni. Education, 38(3) (1989), 1-7.
13. Hayashi.E, Topologies defined by local Properties, Math.Ann., 156 (1964), 205-215.
14. Jankovic.D and Reilly.I.L, On Semi-Separation Properties, Indian J. Pure. Appl.Math., 16(9)(1985),957-964.
15. Jankovic.D and Hamlett.T.R, New topologies from old via Ideals, Amer.Math.Monthly, 97(4) (1990), 295-310.
16. Kannan.K, Narasimhan.D and Chandrasekhara Rao.K., On semi star generalized closed sets in bitopological spaces, Bol.Soc.Paran.Mat.28 (2010), 29-40.
17. Kelly.J.C, Bitopological spaces, Proc.London Math. Society, 13(1963), 71-89.
18. Khan.M and Hamza.M, I_{s*g}-closed sets in Ideal Topological Spaces, Global Journal of Pure and Applied Mathematics, 7(1) (2011), 89-99.
19. Kuratowski.K, Topology I, Warszawa, 1933.
20. Levine.N, Semi-open sets and semi-continuity in topological Spaces, Amer. Math. Monthly, 70(1) (1963), 36-41.
21. Levine.N, Generalized closed sets in topology, Rend. Cric.Mat. Palermo, 19(2) (1970), 89-96.
22. Maheshwari.S.N and Prasad.R, Some new separation axioms, Ann.Soc.Sci.Bruxelles, 89(1975), 395-402.
23. Maheshwari.S.N and Prasad.R, Semi-open sets and Semi-continuous function in bitopological spaces, Math Notae, 26(1977-78), 29-37.
24. Maki.H., Devi.R. and Balachandran.K, Generalized α-*closed* sets in topology, Bull. Fukuoka Univ.Ed.Part III, 42(1993), 13-21.
25. Njastad.O, on some classes of nearly open sets, Pacific J.Math., 15 (1965), 961-970.
26. Renuka Devi.V, Sivaraj.D and Tamizh Chelvam.T, Codense and completely codense ideals, Acta Math.Hungarica., 108(3) (2005), 197-205.
27. Sampath Kumar.S, On decomposition of pairwise continuity, Bull.Cal.Math.Soc.26 (1997), 441-446.
28. Shantha Bose, Semi open sets, semi continuity and semi open mappings in bitopological spaces, Bull.cal.Math.Soc., 73(1981), 237-246.
29. SheikJohn.M and Sundram.P, g*-closed sets in Bitopological spaces Indian J.Pure.Appl. Math., 35(1), 2004, 71-80.
30. Vaidyanathaswamy.R, Set Topology, Chelsea Publishing Company, 1946.
31. Vadivel.A and Swaminathan.A g*p-closed sets in Bitopological spaces, Journal of Advanced studies in Topology 3(1) 2012, 81-88.

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