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A Review on I_{s*g} –Closed Sets in Ideal Topological Spaces

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Abstract: *The purpose of this paper is to review on I_{s*g} -closed sets in ideal topological spaces and some of their properties.*

Keywords: *sg closed, s^*g-closed set, I_{s*g} -closed sets.*

1. Introduction and Preliminaries

An ideal I on a topological space (X,τ) is a collection of subsets of X which satisfies the following properties: (i) $A \in I$ and $B \subseteq A$ implies $B \in I$,(ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. (X, τ, I) represents the topological space with an ideal I. Let $P(X)$ be the set of all subsets of X, a set operator $(\,)^*: P(X) \rightarrow P(X)$, called the local function [20] of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X \,/\, U \cap A \notin I$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(I, \tau)$ in case there is no confusion. X^* is often a proper subset of X. For every ideal topological space (X, τ, I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I, \tau) = \{U \setminus J: U \in \tau$ and $J \in I\}$. It is known in [15] that $\beta(I, \tau)$ is not always a topology on X. A subset A of an ideal space (X, τ, I) is called τ^*-closed [15] or simply *-closed (resp. *-dense in itself) if $A^* \subseteq A$ (resp. $A \subseteq A^*$). A Kuratowski closure operator $cl^*(\,)$ for a topology $\tau^*(I, \tau)$, called the *-topology, is defined by $cl^*(A) = A \cup A^*(\tau, I)$ [31]. M.Khan and M.Hamza [19] introduced the concept of I_{s*g}-closed sets in ideal topological spaces.

Definition: 1.1 A subset A of a topological space is called:

1. semi-open[21] if there exists an open set U in X such that $U \subseteq A \subseteq cl(U)$,
2. α-open [26] if $A \subseteq Int(cl(Int(A)))$,
3. g-closed [22] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X,
4. s^*g-closed [2] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X,
5. $g\alpha$-closed [25] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X,
6. gs-closed [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 1.2.2: A set A of a bitopological space $(X\, \tau_1, \tau_2,)$ is called

1. $\tau_1\tau_2$-semiclosed [24] if there exists an τ_1-closed set F such that τ_2-int $(F) \subseteq A \subseteq F$
2. $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-gclosed set)[11] if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X,
3. $\tau_1\tau_2$-semi generalized closed [10] ($\tau_1\tau_2$-sgclosed) if τ_2 -$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semiopen in X,
4. $\tau_1\tau_2$ - generalized semi closed [9] ($\tau_1\tau_2$-gs closed) if X-A is gs open,
5. $\tau_1\tau_2$-semi star generalized closed [16] ($\tau_1\tau_2$-$s*g$ closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X,
6. $\tau_1\tau_2$-α closed [28] if τ_2- $cl\{\tau_1$-int$[\tau_2$-$cl(A)]\} \subseteq A$,

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7. $\tau_1\tau_2$-g*closed[30] if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-g open,
8. $\tau_1\tau_2$-g*p closed[32] if τ_2-$pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-g open.

Definition 1.2.3: A bitopological space (X, τ_1, τ_2) is called a

1. pairwise $T_{1/2}$-space [11] if every τ_1-g closed set is τ_2-closed and every τ_2-gclosed set is τ_1-closed,
2. pairwise $T*_{1/2}$-space [30] if every $\tau_1\tau_2$-g*closed set is τ_2-closed and every $\tau_2\tau_1$ –g*closed set is τ_1-closed,
3. pairwise T_b-space [9] if every $\tau_1\tau_2$ -gs closed set is τ_2-closed and every $\tau_2\tau_1$ - gs closed set is τ_1-closed,
4. pairwise $T*_p$-space [32] if every $\tau_1\tau_2$ -g*p closed set is τ_2 –closed.

Definition 1.2.4: A subset A of an ideal space (X, τ, I) is said to be I_g- closed [8] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Theorem1.2.5: [16] The arbitrary union of s*g-closed sets A_i, $i \in I$ in a topological space (X, τ) is s*g-closed if the family $\{A_i, i \in I\}$ is locally finite.

Theorem1.2.6: [16] The arbitrary intersection of s*g-open sets A_i, $i \in I$ in a topological space (X, τ) is s*g-open if the family $\{A_i^c, i \in I\}$ is locally finite.

The complement of a semi-open (resp, α-open, I_g.closed) set is semi-closed (resp. α-closed, I_g-open). $SO(X)$ (resp. SC (X,x))represents the collection of all semi-open sets (resp. semi-closed sets containing x) in X.

I_{s*g} –Closed Sets in Ideal Topological Spaces

In this paper, we discuss about I_{s*g} -closed sets in ideal topological spaces and some of their properties.

5.1 Is$_{*g}$ -Closed Sets

Definition 5.1: A subset A of an ideal space (X, τ, I) is said to be I_{s*g} -closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi open in X. The complement of an I_{s*g} -closed set is said to be I_{s*g}-open.

Remark 5.2: Every Is$_{*g}$ -closed set is I_g-closed but the converse is not true in general. To see this, let $X = \{a, b, c, d\}$ with $\tau = \{ \phi, \{a, b\}, X\}$ and $I = \{ \phi, \{a\}\}$. Then $A = \{d\}$ is I_g-closed set but it is not I_{s*g} –closed, since $A^* = \{c, d\}$ and $\{a, b, d\}$ is a semi open set containing A but it is not containing A^*.

Remark 5.3**:**

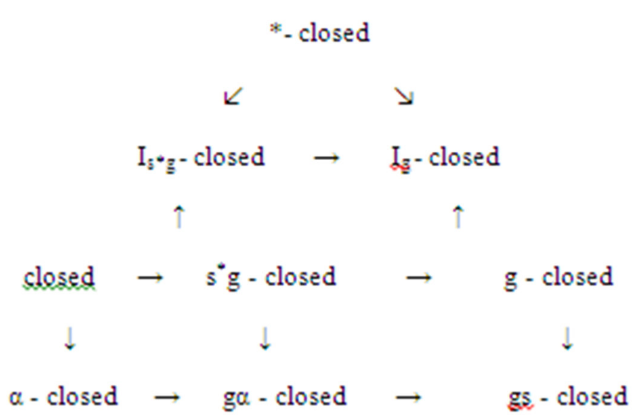
1. Every * -closed set is Is$_{*g}$ -closed but not conversely. To see this, let $X = \{a, b, c\}$ with $\tau = \{ \phi, \{a, b\}, \{c\}, X\}$ and $I = \{ \phi, \{a\}, \{c\}, \{a, c\}\}$.Then $A = \{b, c\}$ is I_{s*g} –closed but it is not * -closed.
2. Every * -closed set is I_g -closed. Converse is true if X is a T_1 -space.
3. In T_1 - space, Is$_{*g}$ -closed sets and I_g - closed sets coincide.

Remark 5.4:

1. I is Is$_{*g}$ -closed in an ideal space (X, τ, I).
2. A^* is Is$_{*g}$ -closed for every subset A of (X, τ, I).

Remark 5.5: (1) The following diagram shows the interrelation between the resulting notion of I$_{s*g}$ –closed sets and related concepts. Reverse implications do not hold.

Diagram - I



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Remark 5.6: In an ideal space (X, τ, I), I_{s*g} - closed sets are generalization of s^*g - closed sets which is itself a generalization of the closed set. An I_{s*g} - closed set may not be s^*g - closed. To see this, let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a, b\}, \{a, b, c\}, X\}$ and $I = \{\phi, \{a\}, \{c\}, \{a, c\}\}$. Then $A = \{a, d\}$ is Is_{*g} - closed set but it is not s^*g - closed. Since $\{a, b, d\}$ is a semi-open set containing A but it is not containing cl(A). An Is_{*g} -closed set is s^*g - closed if $I = \{\phi\}$.

Theorem 5.7: Let (X, τ, I) be an ideal space and A be a nonempty subset of X. Then the following statements are equivalent:

1. A is I_{s*g}- closed;
2. $cl^*(A) \subseteq U$ for every semi open set U containing A;
3. For all $x \in cl^*(A)$, $scl(\{x\}) \cap A \neq \phi$;
4. $cl^*(A)$ - A contains no nonempty semi-closed set;
5. A^* - A contains no nonempty semi-closed set.

Proof. (1) $\Rightarrow$ (2): Let A be an I_{s*g}- closed set. Then clearly $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
(2) $\Rightarrow$ (3): Suppose $x \in cl^*(A)$. If $scl(\{x\}) \cap A = \phi$, then $A \subseteq X - scl(\{x\})$ where $X - scl(\{x\})$ is a semi-open set. By (2), $cl^*(A) \subseteq X - scl(\{x\})$. This contradicts the fact that $x \in cl^*(A)$. Hence $scl(\{x\}) \cap A \neq \phi$. This proves (3).

(3) $\Rightarrow$(4): Suppose $F \subseteq cl^*(A) - A$ where $F \in SC(X, x)$. Since $F \subseteq X - A$ and $\{x\} \subseteq F$. This implies $scl\{x\} \subseteq F$ and $scl(\{x\}) \cap A \neq \phi$. Since $x \in cl^*(A)$, by (3) $scl(\{x\}) \cap A \neq \phi$, a contradiction. This proves (4).

(4) $\Rightarrow$ (5): Assume that $F \subseteq A^* - A$ where $F \in SC(X)$ and $F \neq \phi$. This gives $F \subseteq cl^*(A) - A$. This contradicts (4).

(5) $\Rightarrow$ (1): Let $A \subseteq U$ where $U \in SO(X)$ such that $A^* \not\subset U$. This gives $A^* \cap (X - U) \neq \phi$ or $A^* - [X - (X - U)] \neq \phi$. This gives $A^* - U \neq \phi$. Moreover, $A^* - U = A^* \cap (X - U)$ is semi-closed in X since $A^* = cl(A^*)$ is closed in X by [9, Theorem 2.3 (c)] and $(X - U) \in SC(X)$. Also $A^* - U \subseteq A^* - A$. This gives that $A^* - A$ contains a nonempty semi-closed set. This contradicts (5). This completes the proof.

Theorem 5.8: Let (X, τ, I) be an ideal space and A be a I_{s*g}-closed set. Then following statements are equivalent:

1. A is *- closed set.
2. $cl^*(A) - A$ is a semi-closed set.
3. $A^* - A$ is a semi-closed set.

Proof. (1) $\Rightarrow$ (2): Let A be * -closed set. Then $A^* - A = \phi$. Now $A^* - A = cl^*(A) - A$ gives $cl^*(A) - A = \phi$. This proves that $cl^*(A) - A$ is semi-closed set.

(2) $\Rightarrow$ (3): This is trivial.

(3) $\Rightarrow$ (1): Let $A^* - A$ be a semi-closed set. Now A is Is_{*g} -closed and by Theorem 2.1 (5), $A^* - A$ contains no non empty semi-closed set, therefore $A^* - A = \phi$. This proves $A^* \subset A$ and hence A is *- closed.

Theorem 5.9: In an ideal space (X, τ, I), an I_{s*g}-closed and * -dense set in itself is $s*g$-closed.

Proof. Suppose A is * -dense in itself and Is_{*g} -closed in X. Let U be any semi-open set containing A, then by Theorem 2.1 (2) $cl^*(A) \subset U$. Since A is * -dense in itself, $A \subset A^*$. By [19, Theorem 5] $A^* = cl(A^*) = cl(A) = cl^*(A)$. We get $cl(A) \subset U$ whenever $A \subset U$. This proves that A is $s*g$ - closed.

Corollary 5.10: Let A be a semi-open and I_{s*g} -closed subset of an ideal space (X, τ, I) where I is codense in X. Then A is $s*g$-closed.

Proof. By [19, Theorem 3] **A** is * -dense in itself and hence by Theorem 2.3, A is $s*g$ - closed.

Theorem 5.11: Let (X, τ, I) be an ideal space. If A and B are subsets of X such that $A \subset B \subset cl^*(A)$ and A is I_{s*g} -closed then B is I_{s*g} - closed.

Proof. Since A is I_{s*g}-closed set, by Theorem 2.1(5), $cl^*(A) - A$ contains no nonempty semi-closed set. Since, $A \subset B \subset cl^*(A)$ implies, $cl^*(B) - B \subset cl^*(A) - A$. So $cl^*(B) - B$ contains no nonempty semi-closed set. By Theorem 2.1 (4), B is I_{s*g}-closed.

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Theorem 5.12: Let (X, τ, I) be an ideal space and $A \subseteq X$. Then A is I_{s*g}-closed if and only if $A = F - N$, where F is $*$-closed and N contains no nonempty semi-closed set.

Proof. If A is I_{s*g}- closed set then by Theorem 2.1 (5), $N = A^*- A$ contains no nonempty semi-closed set. Let $F = cl^*(A)$, then F is $*$-closed set and $F - N = (A \cup A^*) - (A^* - A) = A$.

Conversely, let U be any semi-open set in X containing A, then $F - N \subseteq U$ implies $F \cap (X - U) \subseteq F \cap [X - (F \cap N')] = F \cap [(X - F) \cup N] = F \cap N \subseteq N$. By hypothesis $A \subseteq F$ and $F^* \subset F$ as F is $*$- closed gives $A^* \cap (X - U) \subset F^* \cap (X - U) \subset F \cap (X - U) \subset N$, where $A^* \cap (X - U)$ is a semi-closed set. By hypothesis $A^* \cap (X - U) = \phi$ or $A^* \subseteq U$ implies A is I_{s*g} - closed set.

Lemma 5.13: [4, Lemma 2.6] If A and B are subsets of an ideal space (X, τ, I), then $(A \cap B)^* \subseteq A^* \cap B^*$.

Theorem 5.14: Let (X, τ, I) be an ideal space. If A is I_{s*g}- closed and B is $*$ -closed in X, then $A \cap B$ is I_{s*g} - closed.

Proof. Let U be a semi open set in X containing $A \cap B$. Then $A \subseteq U \cup (X - B)$. Since A is I_{s*g} - closed, therefore $A^* \subseteq U \cup (X - B)$ or $B \cap A^* \subseteq U$. Using Lemma 2.1, $(A \cap B)^* \subseteq A^* \cap B^* \subseteq A^* \cap B \subseteq U$ because B is $*$-closed. This proves that $A \cap B$ is I_{s*g} - closed.

Theorem 5.15: Let (X, τ, I) be an ideal space and A be a nonempty subset of X. A is I_{s*g} - open if and only if $F \subseteq \text{int}^*(A)$ whenever $F \subseteq A$ and $F \in SC(X)$.

Proof. Suppose A is I_{s*g}- open set and $F \subseteq A$, where $F \in SC(X)$. Then $X - A \subseteq X - F$. By Theorem 5.1.7 (2), $\text{cl}^*(X - A) \subseteq X - F$. This proves $F \subseteq \text{int}^*(A)$.

Conversely, let U be any semi open set containing $X - A$. Then $X - U \subseteq A$. By hypothesis, $X - U \subseteq int^*(A)$. This implies $cl^*(X - A) \subseteq U$. By Theorem 5.1.7 (1) $X - A$ is I_{s*g} - closed or A is I_{s*g} - open.

Theorem 5.16: Let A be an I_{s*g} -open set in an ideal space (X, τ, I) and $int^*(A) \subset B \subset A$. Then B is I_{s*g} - open.

Proof. Let F be any semi closed set in X contained in B. Then $F \subseteq A$. Since A is I_{s*g} - open. Therefore, by Theorem 5.1.15, $F \subseteq int^*(A)$. But $int^*(A) \subseteq int^*(B)$, implies $F \subseteq int^*(B)$. By Theorem 5.1.15, B is I_{s*g} - open.

Theorem 5.17: Let (X, τ, I) be an ideal space and A be a nonempty subset of X. Then A is I_{s*g}- closed if and only if $A \cup (X - A^*)$ is I_{s*g} -closed.

Proof: Suppose A is I_{s*g}- closed. Let U be a semi-open set such that $A \cup (X - A^*) \subset U$. Then $X - U \subset X - (A \cup (X - A^*)) = A^* - A$. Since A is I_{s*g} - closed, by Theorem 2.1 (5), $X - U = \phi$ and hence $X = U$. Thus X is the only set containing $A \cup (X - A^*)$. This gives $[A \cup (X - A^*)]^* \subset X$. This proves $A \cup (X - A^*)$ is I_{s*g} - closed.

Conversely, let F be any semi-closed set such that $F \subset A^* - A$. Since $A^* - A = X - (A \cup (X - A^*))$. This gives $A \cup (X - A^*) \subset X - F$ and $X - F$ is semi-open. By hypothesis, $(A \cup (X - A^*))^* = X - F$ and hence $F \subset X - A^*$. Since $F \subset A^* - A$ it proves that $F = \phi$ and hence $A^* \subset X - F \in SO(X)$. This proves that A is I_{s*g} - closed.

Theorem 5.18: Let (X, τ, I) be an ideal space and $A \subseteq X$. Then $A \cup (X - A^*)$ is I_{s*g} - closed if and only if $A^* - A$ is I_{s*g} - open.

Proof. Let $A \cup (X - A^*)$ be I_{s*g} - closed. We show that $X - (A^* - A)$ is I_{s*g} - closed. Let U be a semi-open set containing $X - (A^* - A)$. Then $X - U \subseteq A^* - A$. By Theorem 2.1 (5), $X - U = \phi$. Therefore X is the only semi-open set which contains $X - (A^* - A)$ and hence $(X - (A^* - A))^* \subseteq X$. This proves $X - (A^* - A)$ is I_{s*g}- closed or $A^* - A$ is I_{s*g}- open.

Conversely, let $A^* - A$ be I_{s*g}- open. Then $X - (A^* - A) = A \cup X - A^*)$ is I_{s*g} - closed.

Corollary 5. 19: Let (X, τ, I) be an ideal space and $A \subseteq X$. Then A is I_{s*g} - closed if and only if $A^* - A$ is I_{s*g} - open.

Theorem 5.20: Let (X, τ, I) be an ideal space. Then every subset of X is I_{s*g} - closed if and only if every semi open set is $*$ -closed.

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Proof. Suppose every subset of X is I_{s*g} - closed. Let U be a semi-open set then U is Is_{*g} - closed and $U^* \subset U$. Hence U is * -closed.

Conversely, suppose that every semi-open set is *- closed. Let A be a nonempty subset of X contained in a semi-open set U. Then $A^* \subset U^*$ implies $A^* \subset U$. This proves that A is I_{s*g} - closed.

Example 5.21: Consider $\boldsymbol{R}$ the set of all real numbers with the usual topology. If $I = P(\boldsymbol{R})$ then $A^* = \phi$ for every subset A of X or $A^* \subset A$. This proves that A is * -closed.

Definition 5.22: The intersection of all semi-open subsets of a space X containing set A is known as semi kernel of A and denoted by $s\ ker(A)$.

Lemma 5.23: A *- dense in itself subset A of a space X is I_{s*g} - closed if and only if $A^* \subseteq s\ ker(A)$.

Proof. Assume that an I_{s*g} - closed set A is a * -dense in itself. Then by [19, Theorem 5], $A^* = cl\ (A)$.But $A^* \subseteq \cap\{G{:}A \subseteq G$ and $G \in SO(X)\} = s\ ker\ (A)$. The converse is trivial.

Lemma 5.24: [8, Lemma 2] every singleton $\{x\}$ in a space X is either no-where dense or preopen.

Theorem 5.25: Arbitrary intersection of *- dense in itself, I_{s*g} - closed sets in an ideal space (X, τ, I) is I_{s*g} - closed.

Proof. Let $\{A_\alpha{:}\ \alpha \in \Omega\}$ be an arbitrary collection of * -dense, Is_{*g} - closed sets in an ideal space (X, τ, I) and let $A = \cap A_\alpha$. Let $x \in A^*$. In view of Lemma 2.3, we consider the following two cases.

Case 1: $\{x\}$ is no-where dense. If $x \notin A$, then for some $j \in \Omega$, we have $x \notin A_j$. Since no-where dense subsets are semi closed [3, Theorem 1.3], therefore $x \notin s\ ker(A_j)$. Again by Lemma 2.2. $A_j^* \subseteq s\ ker(A_j)$. Since A_j is * -dense in itself, Is_{*g} - closed implies $x \in A^* = cl(A) \subseteq cl(A_j) \subseteq s\ ker(A_j)$. By contradiction $x \in A$ and hence $x \in s\ ker(A)$. This proves that $A^* \subseteq s\ ker(A)$ and hence by Lemma 2.2, A is I_{s*g} - closed.

Case 2: $\{x\}$ is pre open. Let $F = int(cl(\{x\}))$. Assume that $x \notin s\ ker(A)$. Then, there exist a semi closed set C containing x such that $C \cap A = \phi$. Now by [3, Theorem 1.2] $x \in F = int(cl(\{x\})) \subseteq int(cl(cl)) \subseteq C$. Since F is an open set containing x and $x \in cl(A) = A^*$, therefore, $F \cap A \neq \phi$. Since $F \subseteq C$ therefore $C \cap A \neq \phi$. A contradiction. Hence $x \in s\ ker(A)$. By Lemma 2.2, A is I_{s*g} - closed.

Lemma 5.26: Let $\{A{:}\ i \in \Omega\}$ be a locally finite family of sets in an ideal space (X, τ, I). Then $\cup_{i \in \Omega} A_i^*(I) = (\cup_{i \in \Omega} A_i)^*\ (I)$.

Theorem 5.27: Let (X, τ, I) be an ideal space. If $\{A_i{:}\ i \in \Omega\}$ is a locally finite family of sets and each A_i is I_{s*g} - closed in (X, τ, I). Then $\cup_{i \in \Omega} A_i$ is Is_{*g} - closed.

Proof. Let $\cup_{i \in \Omega} A_i \subseteq U$ where U is semi open set in X. Since for each i, A_i is I_{s*g} - closed, $A_i^* \subseteq U$ for each $i \in \Omega$. Hence $\cup_{i \in \Omega} A_i^* \subseteq U$. Using Lemma 2.4, $(\cup_{i \in \Omega} A_i)^* \subseteq U$. Hence $\cup_{i \in \Omega} A_i$ is Is_{*g} - closed.

Theorem 5.28: Union of two $\mathbf{Is}_{*g}$ - closed set is I_{s*g} - closed.

Proof. Let A, B be I_{s*g} - closed sets and W be a semi-closed set such that $A \cup B \subseteq W$. This implies $A^* \subseteq W$ and $B^* \subseteq W$. This implies $A^* \cup B^* = (A \cup B)^* \subseteq W$. This proves that $A \cup B$ is I_{s*g} - closed set.

Example 5.29: Let $X = N$ and $\boldsymbol{\tau}$ be the cofinite topology. Let $\{A_n : A_n = \{2, 3, \ldots\ldots, n+1\}, n \in N\}$ be a collection of I_{s*g} - closed sets in X. Then $\cup_{n \in N} A_n = N \setminus \{1\} = A$ (say) having a finite complement is open and hence semi open but not closed. As $A^* = cl(A) = N \not\subset A$ for $I = \phi$, gives that A is not I_{s*g} - closed but $A^* = \phi \subseteq A$ for $I = P(X)$. In this case arbitrary union of I_{s*g} - closed sets is I_{s*g} - closed.

Theorem 5.30: Every open set is I_{s*g} - open.

Proof. Let U be an open set. We need to show U is I_{s*g} - open. For this we show that $X - U$ is I_{s*g} - closed. Let $X - U \subset G$ where $G \in SO(X)$. Since $X - U$ is closed. So by [9, Theorem 2.3] $(X - U)^* \subseteq cl(X - U) = X - U$ or $(X - U)^* \subseteq (X - U) \subset G$. This proves that $X - U$ is I_{s*g} - closed or U is I_{s*g} - open.

Definition 5.31: A space X is s^*- normal, if for each pair of disjoint semi-closed sets A and B, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Theorem 5.32: Let (X, τ, I) be an ideal space where I is completely co-dense. Then the following statements are equivalent:

1. X is s^*- normal.
2. For any disjoint semi closed sets A and B, there exist disjoint I_{s*g} - open sets U and V containing A and B respectively.
3. For any semi closed set A and semi open set V containing A there exists an I_{s*g} - open set U such that $A \subset U \subset cl^*(U) \subset V$.

Proof. (1) $\Rightarrow$ (2) This proof follows from the fact that every open set is Is_{*g} - open set.

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(2) $\Rightarrow$ (3) Suppose A is semi-closed and V is semi-open set containing A. Since A and $X - V$ are disjoint semi-closed sets, there exist disjoint I_{s*g} - open sets U and W such that $A \subset U$ and $X - V \subset W$. Since $X - V$ is semi-closed and W is I_{s*g}- open. By Theorem 2.7, $X - V \subset int^*(W)$ and hence X - $int^*(W) \subset V$.Again $U \cap W = \phi$ implies $U \cap int^*(W) = \phi$ and hence $cl^*(U) \subset X - int(W) \subset V$. Thus U is the required Is_{*g}- open set. This implies $A \subset U \subset cl^*(U) \subset V$.

(3) $\Rightarrow$ (1) Let A and B be two disjoint semi-closed subsets of X. By hypothesis there exists an I_{s*g}- open set U such that $A \subset U \subset cl^*(U) \subset X$ - B. Since U is I_{s*g} - open set and $A \subset U$, by Theorem 2.7, $A \subset int^*(U)$. Since I is completely co-dense, by [19, Theorem 6], $\tau^* \subset \tau^{\alpha}$ and so $int^*(U)$ and $X - cl(U) \in \tau^{\alpha}$. Hence $A \subset int^*(U) \subset int\ (cl\ (int(int^*(U)))) = G$ and $B \subset X$- $cl^*(U) \subset int(cl(int(X$- $cl^*(U)))) = H$. Hence, G and H are required disjoint open sets containing A and B respectively. This proves (1). This completes the proof.

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