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| ISBN | 978-81-929866-6-1 |
| Website | icsscet.org |
| Received | 25 – February – 2016 |
| Article ID | ICSSCET032 |

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|-----------------|---------------------|
| VOL | 02 |
| eMail | icsscet@asdf.res.in |
| Accepted | 10 - March – 2016 |
| eAID | ICSSCET.2016.032 |

An Inventory Model for a Closed Loop Supply Chain Considering Product Life Cycle in a Discrete Period

A Sasikumar¹, K Natarajan² and M Ramasubramaniam³

¹Karpagam Institute of Technology, ²P.S.G College of Technology, ³ Loyola Institute of Business Administration

Abstract- This paper focus on finding the optimum quantity of Product and Parts at various site in a Closed Loop supply chain (CLSC) system during the product life cycle. The experience from the reverse logistics industries show that three major return-recovery pairs namely commercial returns, End-of use returns and end of life returns are important. In this paper, a network consists of manufacturer, distributor, retailer, customers and for returns repair, collection site, repair site, disassembly site, recycling site and disposal site are considered and a model is developed for maximizing the profit in a CLSC system. The purpose of this model is to find the optimal inventory positioning at different stages of CLSC during different phases of lifecycle. The model is configured as mixed integer linear programming. A sensitivity analysis of results show that the decision variables involved in inventory positioning at various sites depend on phase of product life cycle.

Keywords: Closed loop supply chain (CLSC), Product Life cycle (PLC), Mixed- Integer Linear Programming (MILP), Inventory

I. INTRODUCTION

The Closed Loop Supply Chain (CLSC) is a process in which the manufacturer systematically accepts the shipped products or parts from the point of consumptions for possible repair, recycle, remanufacture and proper disposal. A CLSC effectively manages resources by suitably directing the flow of parts and products for remanufacturing, recycling or disposal sites. Thus, CLSC is of practical interest for supply chain managers and executives.

The importance of CLSC is multi-fold when considering a product's lifecycle along with the above said issues. For instance, product life cycle of certain products such as mobile phone, computers, printer cartridge, medicine, and electronic components are very short. Within a product's lifecycle, the industries have to effectively manage resources to maximize the profit.

The shorter lifespan of product's lifecycle influences the way inventory is positioned across the CLSC. This is further complicated because of sporadic end-of-life, end-of-use and commercial returns in such CLSC.

Returns of the product may be in different forms in the quantity and quality and it can also be categorized in to three types. First is commercial returns of the product which can be repaired, refurbishing in the repair site. Second is end- of- use returns parts after disassembly of the products these usable parts can be hold in the part inventory and finally end- of- life returns parts can be recycled and some of them can also be disposed.

Consequently, optimal product-mix and inventory positioning is of paramount important in a CLSC under product lifecycle. Since manually solving this problem is an arduous task, in this paper we propose a mathematical model to maximize the profit of CLSC system by determining the optimal inventory and cost associated at various sites with the demand and returns variations along the four phases of PLC.

This paper is prepared exclusively for International Conference on Systems, Science, Control, Communication, Engineering and Technology 2016 [ICSSCET 2016] which is published by ASDF International, Registered in London, United Kingdom under the directions of the Editor-in-Chief Dr T Ramachandran and Editors Dr. Daniel James, Dr. Kokula Krishna Hari Kunasekaran and Dr. Saikishore Elangovan. Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage, and that copies bear this notice and the full citation on the first page. Copyrights for third-party components of this work must be honoured. For all other uses, contact the owner/author(s). Copyright Holder can be reached at copy@asdf.international for distribution.

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Cite this article as: A Sasikumar, K Natarajan, M Ramasubramaniam. "An Inventory Model for a Closed Loop Supply Chain Considering Product Life Cycle in a Discrete Period". *International Conference on Systems, Science, Control, Communication, Engineering and Technology 2016*: 165-176. Print.

II. Literature Review

A number of authors have addressed the profitability problem in a CLSC. In a case based approach in a battery recycling industry, Kannan et.al [3] developed a model in multi echelon, multi product, multi period in a CLSC network. The model considers only recycling of end-of-life products. Saman Hassanzadeh Amin and Guoqing Zhang [7] proposed a model by configuring a different remanufacturing process to find optimal quantity of products and parts in the CLSC network. The performance of the model is analysed for multiple products within a single period and validated through computational testing.

In another case study research, Sasikumar et.al [8] in a truck tyre remanufacturing shows that for maximizing the profit in CLSC, the decisions related to the number of facilities, their location, and the allocation of the corresponding product flows are important. For the same industry and also including plastic goods manufacturing, Kannan et.al [4] integrated the multi echelon distribution inventory model for forward and reverse network by minimizing the cost to increase profitability.

Addressing the uncertainty in the returns, Subramanian et.al [10] integrated the forward and reverse supply chain with proper allocation-location model for the warehouse. In this model they have considered only single product, single period with constant demand and uncertain in returns. Extending the model, Jianmai Shi et.al [2] developed a model to maximise the manufacturer expected profit by determining the production quantities of a new product, the quantities of remanufactured product and the acquisition prices of the used product within capacity constraints.

While the type of returns and uncertainties associated with them are important, the problem becomes complex when we consider the product lifecycle. There is some research which throws light on this aspect. Che-Fu Hsueh [1] investigates, inventory control policies in a manufacturing/remanufacturing system with PLC assuming demand rate and return rate as a random variable. The result of this investigation shows that different inventory control policies should be adopted during different phase of the PLC.

In addition to this, the inventory policy characterization for single product over the entire PLC with various set up costs is studied by Sebnem Ahiska, and Russell, E.King [9]. In the study the optimal or near-optimal policy characterizations with practical structure is determined for every life cycle stage under several setup cost configurations. They devise policies which control the parameters in CLSC under deferred sales in a PLC situation.

Along with the issue of PLC, capacity planning issues also have been addressed Patroklos Georgiadis et.al [5]. The result of the study shows that collection and remanufacturing capacity policies are insensitive to the total product demand.

Thus, although the issue of returns and PLC have been addressed in a CLSC, there is a need for a comprehensive model which integrates the three different returns with the PLC during its various stages and finds optimal inventory positioning under different demand conditions. Hence, in this paper we develop a model for maximizing the profit of CLSC by determining the optimal inventory and cost associated at various sites with the demand and returns variations along different phases of PLC.

III. Problem Definition

From Industry point of view, there are various types of CLSC network configurations are possible. Among these, we propose a generalized form of CLSC framework. In this study, the framework of reverse logistics consists of a manufacturer, collection site, repair site, disassembly site, disposal site and recycling sites as shown in the Figure 1. After using the products, some of the customers return the used products. The returned products are then collected during different phase of PLC at collection site and are segregated in to two types of returns. One, commercial returns of the returns products which are sent to the repair site for refurbishing and small repair. Second, the end of life returns which are taken to the disassembly sites for disassemble the product into parts. The unused parts can be disposed to the disposed site and the end-of-use parts can be sent to recycling site for processing and the good parts are taken to part inventory at different phase of PLC.

Here the final end period inventory of a product at manufacturer, distributor and retailer are considered so that it could be the initial inventory for subsequent periods. In addition, unit inventory holding cost, shipment cost, set up cost and capacity constraints of repair site, disassembly site and recycling site are taken in to account. If the demand and inventory of the products are more than units returned, then the manufacturer should produce new products at the manufacturing site. The main objective of the model is to maximization of profit by determining the optimal quantity and inventory of products and parts during different phase of PLC.

To maintain model parsimony, the framework of Saman Hassanzadeh Amin and Guoqing Zhang [7] is considered without external suppliers. Further, we assume that if the demand of the product during different phase of PLC is more than the returned products then manufacturer has capacity to produce new products.

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IV. Model Formulations and Assumptions

The Indices, Parameters and its associated decision variables and the mathematical model formation of the proposed closed loop supply chain are specified below. For computation purpose, the various input data are taken from the literature Saman Hassanzadeh Amin and Guoqing Zhang [7].

The assumptions involved in this model are as follows:

- Reverse Logistics industry can use its experience or the historical data of product to predict the phase length of PLC. In this paper, without loss of generality, different phases of a PLC are demarcated according to the demand and return. Each phase is described as follows:
 - 1) Introduction: Demand remains at a low and the returns are rarely seen and can be ignored.
 - 2) Growth: Demand begins to increase linearly and some returns will emerge as percentage of the demand.
 - 3) Maturity: Demands remains in a steady state, without increase or decrease but the percentage of returned products will be more than the previous phase
 - 4) Decline: Demands starts to decrease linearly with greater percentage of returns
- The lead time of delivery for various site are ignored
- The process involved in the repair site, disassembly site, recycling site of return product are done immediately.
- If the quantity of the product is not enough to meet requirements by the manufacturer then manufacturer should produce inside the manufacturing site.
- Maximum capacity of manufacturer, disassembly, repair, and recycling sites are known
- The capacity of collection site is unlimited.
- The initial inventory of the manufacturer is known
- The final period of the inventory of manufacturer, distributor and retailer also known so that it will act as opening inventory for next period.

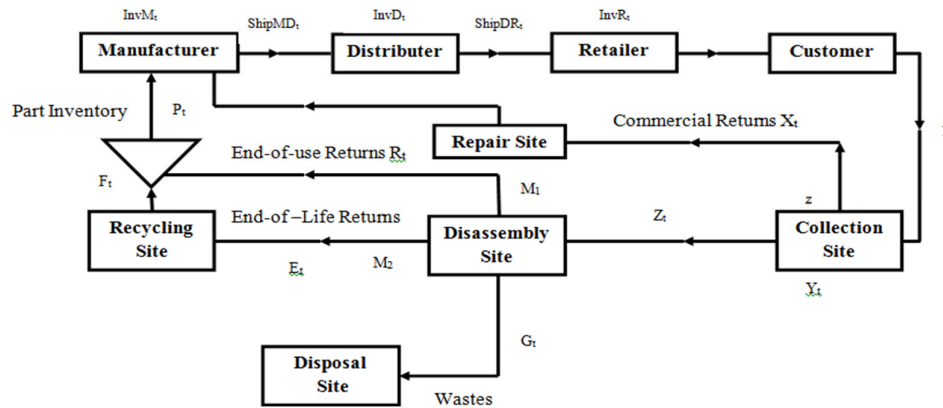


Figure.1 Proposed framework of CLSC

The Indices, Parameters and its associated decision variables and the mathematical model formation of the proposed closed loop supply chain are specified below.

- t Set of period in different phases of PLC, $t = 1, 2, \dots, T$
- X_t Product to be repaired at different phases of PLC
- P_t Product obtained from part inventory at different phases of PLC
- Y_t Product collected in collection site at different phases of PLC
- Z_t Product to be disassembled at different phases of PLC
- Mfg_t Product to be manufactured in the manufactured site at different phases of PLC
- E_t Part are obtained in the disassembly site at different phases of PLC
- R_t End-of-use returns part at different phases of PLC
- G_t Parts to be disposed at different phases of PLC
- F_t Part to be recycled in recycling site at different phases of PLC
- $InvM_t$ Inventory of Product at the manufacturing site at different phases of PLC
- $InvR_t$ Inventory of Product at the retailer at different phases of PLC
- $InvD_t$ Inventory of Product at the distributor at different phases of PLC
- $ShipMD_t$ Shipment of Product from manufacturer to distributor at different phases of PLC
- $ShipDR_t$ Shipment of Product from distributor to retailer at different phases of PLC

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| | |
|----------------|--|
| U _t | Binary variable for set up of recycling site for part at different phases of PLC |
| V _t | Binary variable for set up of disassembly site at different phases of PLC |
| W _t | Binary variable for set up of repair site at different phases of PLC |
| S | Unit selling price for the product |
| A | Resource usage to produce one unit of product |
| y | Unit direct manufacturing cost of Product |
| e | Resource usage to repair one unit of product |
| C | Max Capacity of repair site for product |
| D _t | Demand for product j at period t |
| d | Unit repair cost of Product |
| f | Set-up cost of disassembly site for product |
| g | Set-up cost of repair site for product |
| B | Max capacity of disassembly site to disassemble part |
| h | Unit disassembly cost for part |
| m | Unit disposal cost for part |
| r | Resource usage to disassemble one unit of part |
| q | Unit requirements for part to produce one unit of product |
| n | Unit recycling cost for part in recycling site |
| o | Set-up cost of recycling site for part |
| s | Resource usage to recycle one unit of part in recycling site |
| O | Max capacity of recycling site to recycle part |
| HR | Unit Inventory holding cost at retailer of Product |
| HD | Unit Inventory holding cost at Distributor of Product |
| HM | Unit Inventory holding cost at Manufacturer of Product |
| SH | Unit Shipment cost of product |
| I | Inventory of a product at Manufacturing site at period t=1 |
| F1 | Inventory of a product at retailer at the end Period t |
| F2 | Inventory of a Product at distributor at the end Period t |
| F3 | Inventory of a Product at Manufacturer at the end Period t |
| MD | Max capacity of truck to travel from Manufacturer to Distributor |
| DR | Max capacity of truck to travel from Distributor to Retailer |
| z | Max percent of commercial returns |
| M | A big number |
| N | Max percent of total returns at different phases |
| M1 | Max percent of end-of-use returns |
| M2 | Max percent of end-of-life returns |
| L | Max number of recycling sites |
| A | Max capacity of the manufacturer plant |

Max Z

$$\begin{aligned}
 & \sum_{t=1}^T S(X_t + P_t + Mfg_t) - \sum_{t=1}^T y(Mfg_t) - \sum_{t=1}^T h(E_t) - \sum_{t=1}^T f(V_t) - \sum_{t=1}^T d(X_t) \\
 & - \sum_{t=1}^T m(G_t) - \sum_{t=1}^T g(W_t) - \sum_{t=1}^T n(F_t) - \sum_{t=1}^T o(U_t) - \sum_{t=1}^T HR(InvR_t) - \sum_{t=1}^T HD(InvD_t) - \sum_{t=1}^T HM(InvM_t) \\
 & - \sum_{t=1}^T SH(ShipMD_t + ShipDR_t) \tag{1}
 \end{aligned}$$

Subject to

$$q(P_t) = R_t + F_t \tag{2}$$

$$F_t + G_t + R_t = E_t \tag{3}$$

$$q(Z_t) = E_t \tag{4}$$

$$P_t + X_t = Mfg_t \tag{5}$$

$$R_t \leq M_1 E_t \quad (6)$$

$$F_t \leq M_2 E_t \quad (7)$$

$$G_t \leq (1 - M_1 - M_2) E_t \quad (8)$$

$$Mfg_t \leq I \quad (9)$$

$$InvR_t \leq F_1 \quad (10)$$

$$InvD_t \leq F_2 \quad (11)$$

$$InvM_t \leq F_3 \quad (12)$$

$$InvR_{t-1} + ShipDR_{t-1} - D_t = InvR_t \quad (13)$$

$$InvD_{t-1} + ShipMD_{t-1} - ShipDR_t = InvD_t \quad (14)$$

$$Mfg_t + InvM_{t-1} - ShipMD_t = InvM_t \quad (15)$$

$$InvM_{j_{t-1}} \geq P_{j_{t-1}} + X_{j_{t-1}} + Mfg_{t-1} \quad (16)$$

$$ShipMD_t \leq MD \quad (17)$$

$$ShipDR_t \leq DR \quad (18)$$

$$X_t + Z_t = Y_t \quad (19)$$

$$a(Mfg_t) \leq A \quad (20)$$

$$r(E_t) \leq B \quad (21)$$

$$s(F_t) \leq O(U_t) \quad (22)$$

$$e(X_t) \leq C \quad (23)$$

$$X_t \leq zY_t \quad (24)$$

$$Z_t \leq (1 - z)Y_t \quad (25)$$

$$Y_t \leq N(D_t) \quad (26)$$

$$\sum_{t=1}^T U_t \leq L \quad (27)$$

$$Z_t \leq M(V_t) \quad (28)$$

$$X_t \leq M_1(W_t) \quad (29)$$

$$U_t, V_t, W_t \in \{0, 1\} \quad (30)$$

$$X_t, P_t, Y_t, Z_t, Mfg_t, E_t, F_t, R_r, G_t$$

$$InvM_t, InvR_r, InvD_r, ShipMD_t, ShipDR_t \geq 0 \quad (31)$$

The objective function (1) maximizes the total profit for the manufacturer. The first term in the objective function represents the total selling profit of the product. It includes the repaired product with new product and shortage of the product to be produced in the manufacturing site during different phases of PLC. Second term represents the unit direct manufacturing cost multiplied by the amount of manufactured item in the manufactured site at different phases of PLC. Third and fourth terms of the expression represent the unit disassembly cost and set up cost for disassembly site at different phases of PLC. The fifth, sixth and seventh term represents the unit repair cost, disposal cost and set up cost for repair in the repair site. The next two terms of the objective function includes the cost related to unit recycling cost and set up cost of recycling at recycling site. Finally the last four terms of the expressions represents the inventory cost associated with the retailer, distributor, manufacturer and shipment cost from the manufacturer to distributor and from distributor to retailer respectively.

Constraint (2) ensures that the number of recycled parts is equal to the number of manufactured parts and number of end-of-use parts. The relationship between number of disassemble parts equal to the summation of number of recycling parts and end-of-use parts and disposal parts are presented in Constraint (3). Constraint (4) ensures the relationship between parts and products in disassembly site. The sum of parts from part inventory and the products from repair site is equal to manufacturing product is represented in constraints (5).

Constraints (6) to (8) show the Percent of end-of-use returns and end-of-life returns. Initial inventory at Manufacturer site at period 1 is represented in constraints (9). Final inventory of the product at the end period of retailer, distributor and manufacturer are considered in the Constraint (10) to (12). The inventory and shipment restriction at period t and previous period t-1 for retailer, distributor and manufacturer are enforced from the constraint (13) to (16). Capacity constraints of truck from manufacturer to distributor and from distributor to retailer are ensured in the constraints (17) and (18). Besides the Constraints (19) represents that the collected products are sent to repair or disassembly site.

Constraint (20) to (23) reflects the maximum capacity of manufacturer, disassembly, recycling and repair sites. Constraint (24) and (25) represents maximum percent of commercial returns.

The maximum percentage of total returns at the collecting site at various phases of PLC is considered in constraints (26). In addition to this, the limitation of recycling is represented in constraints (27). The constraints (28) and (29) ensure the units of returned products to be disassembled and repaired at different phases of PLC. Finally, the decisions variables are defined in the constraints (30) and (31). The proposed model is in the form of Mixed Integer linear programming problem and solved by IBM ILOG CPLEX OPL studio. The obtained results are validated through computational testing and sensitivity analysis.

V. Computational Results

In this section for testing the model numerical examples are considered with appropriate input parameters. Based on the assumptions for different phase of PLC the demand and Percentage of return product are quoted as 1500 product of demand with zero return at introduction phase. In growth phase demand increase linearly of 500 product and the 30 percent returns and at maturity phase the demand is steady as 4000 with 50 percent of return. During the decline phase demand starts decreases linearly of 500 Product with 80 percent of returns. Apart from these parameters the final inventory at the end period for manufacturer, distributor and retailers are known. Here the total length of PLC chosen as t=16 periods in that first 3 periods is for introduction phase, next 5 periods is for growth phase and the remaining 5 periods for maturity and last 3 periods is for decline phase. The final end period inventories F1=100, F2= 200, F3= 300 of retailer, distributor, manufacturer and shipments capacity are quoted.

According to results of the proposed model, returned products are collected in collection site with various percent for different phases of PLC are as follows: (i) In introduction phase zero percent returns. (ii) At growth phase 30 percent (iii) In Maturity phase 50 percent of returns and (iv) Finally a 70 percent of returns were considered in the decline phase of PLC. Among the returns, 60 percent of products are utilized as commercial returns which are to be repaired at repair site. And remaining 40 percent are sent to the disassembly site.

After disassembly of product into parts in the disassembly site, 30 percent of parts are taken as End-of –use returns to the part inventory and another 30 percent of parts are considered as End-of-life returns. These End-of –life returns parts are sent to recycling site. Remaining 40 percent of parts are sent to the disposed sites. The other required input parameters are written in Appendix-I. In this paper, the optimal solution of the mixed integer program is obtained by IBM ILOG CPLEX OPL studio (version 12.5)

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The decision variables for the quantity of product and parts are various sites for different phases of PLC is shown in the Table.1. The inventory at retailer, distributor and Manufacturer and the shipment from Distributor to retailer and from manufacturer to distributor for different phase of PLC is shown in the Table.2. The various cost associated with the objective function are summarized in the Table. 3

Table1. Product and parts at various sites

| Phases | Period | Demand | Returns | Product from repair site | Product from Part Inventory | Product to be manufactured in manufacturing site | Parts obtained in disassembly site | Parts from end-of-use returns | Parts to be recycled | Parts to be disposed |
|--------------|--------|--------|---------|--------------------------|-----------------------------|--|------------------------------------|-------------------------------|----------------------|----------------------|
| | t | D_t | Y_t | X_t | P_t | Mfg_t | E_t | R_t | F_t | G_t |
| Introductory | 1 | 1500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 2 | 1500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 3 | 1500 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Growth | 4 | 2000 | 600 | 360 | 144 | 504 | 480 | 144 | 144 | 192 |
| | 5 | 2500 | 750 | 450 | 180 | 630 | 600 | 180 | 180 | 240 |
| | 6 | 3000 | 900 | 540 | 216 | 756 | 720 | 216 | 216 | 288 |
| | 7 | 3500 | 1050 | 630 | 252 | 882 | 840 | 252 | 252 | 336 |
| Maturity | 8 | 4000 | 1175 | 705 | 282 | 987 | 940 | 282 | 282 | 376 |
| | 9 | 4000 | 1175 | 705 | 282 | 987 | 940 | 282 | 282 | 376 |
| | 10 | 4000 | 1150 | 690 | 276 | 966 | 920 | 276 | 276 | 368 |
| | 11 | 4000 | 1125 | 675 | 270 | 945 | 900 | 270 | 270 | 360 |
| | 12 | 4000 | 1050 | 630 | 252 | 882 | 840 | 252 | 252 | 336 |
| | 13 | 4000 | 875 | 525 | 210 | 735 | 700 | 210 | 210 | 280 |
| Decline | 14 | 3500 | 600 | 360 | 144 | 504 | 480 | 144 | 144 | 192 |
| | 15 | 3000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 16 | 2500 | 1525 | 915 | 366 | 1281 | 1220 | 366 | 366 | 488 |

Table 2. Inventory and shipment variables

| Phases | Period | Inventory at Retailer | Inventory at Distributor | Inventory at Manufacturer | Shipment from Manufacturer to Distributor | Shipment from Distributor to Retailer |
|--------------|--------|-----------------------|--------------------------|---------------------------|---|---------------------------------------|
| | t | $InvR_t$ | $InvD_t$ | $InvM_t$ | $ShipMD_t$ | $ShipDR_t$ |
| Introductory | 1 | 0 | 22000 | 5233 | 0 | 12000 |
| | 2 | 10500 | 19500 | 4233 | 1000 | 2500 |
| | 3 | 11500 | 18000 | 3233 | 1000 | 2500 |
| Growth | 4 | 12000 | 16500 | 2737 | 1000 | 2500 |
| | 5 | 12000 | 15000 | 2367 | 1000 | 2500 |
| | 6 | 11500 | 13500 | 2123 | 1000 | 2500 |
| | 7 | 10500 | 12000 | 2005 | 1000 | 2500 |
| | 8 | 9000 | 10500 | 1992 | 1000 | 2500 |
| Maturity | 9 | 7500 | 9000 | 1979 | 1000 | 2500 |
| | 10 | 6000 | 7500 | 1945 | 1000 | 2500 |
| | 11 | 4500 | 6000 | 1890 | 1000 | 2500 |
| | 12 | 3000 | 4500 | 1772 | 1000 | 2500 |
| | 13 | 1500 | 3000 | 1507 | 1000 | 2500 |
| Decline | 14 | 500 | 1500 | 1011 | 1000 | 2500 |
| | 15 | 0 | 0 | 19 | 992 | 2500 |
| | 16 | 0 | 0 | 300 | 1000 | 992 |

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Table 3. Cost related parameters

| Phases | Period | Disassembly cost of product | Disassembly cost of Parts | Disposal cost | Inventory cost at Distributor | Inventory cost at Manufacturer | Inventory cost at Retailer | Repair cost | Shipment cost | Recycling cost |
|--------------|--------|-----------------------------|---------------------------|---------------|-------------------------------|--------------------------------|----------------------------|-------------|---------------|----------------|
| Introduction | 1 | 0 | 0 | 0 | 66000 | 104666 | 0 | 0 | 36000 | 0 |
| | 2 | 0 | 0 | 0 | 58500 | 8466 | 52500 | 0 | 10500 | 0 |
| | 3 | 0 | 0 | 0 | 54000 | 6466 | 57500 | 0 | 10500 | 0 |
| Growth | 4 | 5 | 1920 | 576 | 49500 | 5457 | 60000 | 365 | 10500 | 292 |
| | 5 | 5 | 2400 | 720 | 45000 | 4734 | 60000 | 455 | 10500 | 364 |
| | 6 | 5 | 2880 | 864 | 40500 | 4246 | 57500 | 545 | 10500 | 436 |
| | 7 | 5 | 3360 | 1008 | 36000 | 4010 | 52500 | 635 | 10500 | 508 |
| Maturity | 8 | 5 | 3750 | 1128 | 31500 | 3984 | 45000 | 710 | 10500 | 568 |
| | 9 | 5 | 3760 | 1128 | 27000 | 3958 | 37500 | 710 | 10500 | 568 |
| | 10 | 5 | 3680 | 1104 | 22500 | 3890 | 30000 | 695 | 10500 | 556 |
| | 11 | 5 | 3600 | 1080 | 18000 | 3780 | 22500 | 680 | 10500 | 544 |
| Decline | 12 | 5 | 3360 | 1008 | 13500 | 3544 | 15000 | 635 | 10500 | 508 |
| | 13 | 5 | 2800 | 840 | 9000 | 3014 | 7500 | 530 | 10500 | 424 |
| | 14 | 5 | 1920 | 576 | 4500 | 2022 | 2500 | 365 | 10500 | 292 |
| | 15 | 0 | 0 | 0 | 0 | 38 | 0 | 0 | 10476 | 0 |
| | 16 | 5 | 4880 | 1464 | 0 | 600 | 0 | 920 | 5976 | 736 |

VI. Sensitivity Analysis

A sensitivity analysis is performed to validate the proposed model by taking the average values of product and parts at different phase of PLC with various decision variables. The Figure.2 represents the average demand and returns at different phase of PLC. The decision variable number of product to be manufactured at manufacture site is lower than the part inventory and commercial return of the product at all the phases of PLC as illustrated in the figure.3. Hence the manufacturer can take appropriate decision at all the phases of PLC by considering the percent of return product obtained as commercial return at repair site and part inventory. For the given 30 percent of parts returns as End –of life returns and End of use returns which are disassembled in the disassembly site are much lower than the 40 percent of disposed parts and the total number of parts at disassembly site for all the phases of PLC is observe in the figure.4.

Figure2. Average Demand and Returns at different phase of PLC

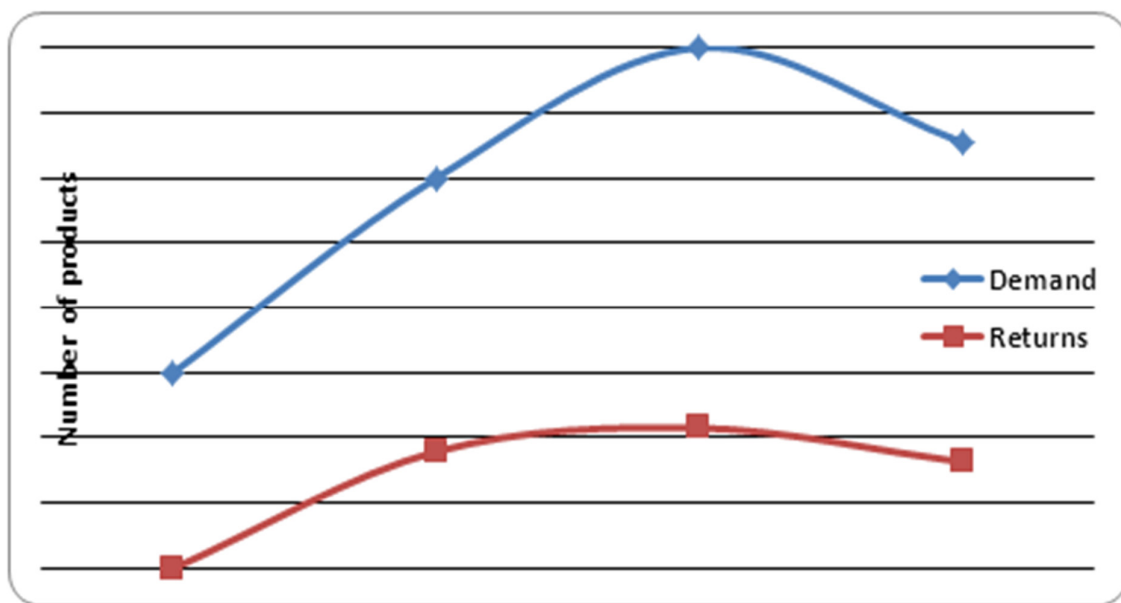


Figure 3. Commercial returns of the product Vs Manufactured product at different phase of PLC

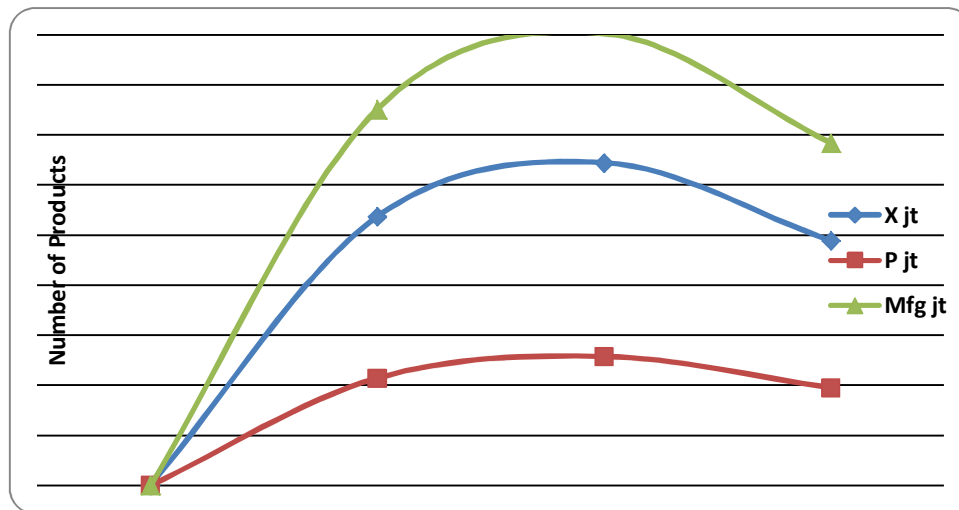
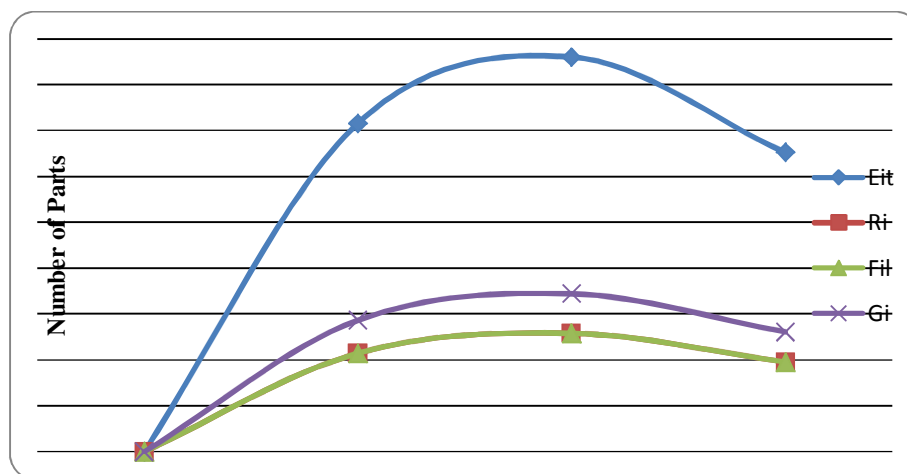


Figure 4. Disassembly parts as end-of-life returns, end-of-use returns and disposed parts at different stage of PLC



The inventory parameter at manufacturer, retailer and distributor are illustrated in the Figure.5 which indicated the average inventory of product at distributor is more than the inventory of retailer and manufacturer for all the phases of the PLC. It has been observed that the average inventory of distributor is linearly decreasing throughout the product life cycle. The average inventory at retailer is linearly increasing from introduction to growth stage after which there is a decrease in inventory levels throughout the life cycle of the product. But the manufacturer inventory is lower than the inventory of retailer and distributor for the entire life of the PLC. Therefore the cost parameters involved in keeping the inventory at retailer, distributor and manufacturer are very important. Hence the cost analysis is carried out for sensitivity and the average results obtained at different phase of PLC is shown in the figure.6. Here the following inventory cost at various phase of PLC is discussed:

- Introduction stage: The average cost of inventory at distributor is very high when compared with inventory cost at manufacturer and retailer.
- Growth stage: The average inventory cost of retailer is more than the cost of distributor and manufacturer.
- Maturity stage: Average inventory cost of retailer is more than the cost of distributor. When compared retailer and distributor the average inventory cost at manufacturer is negligible.
- Decline stage: Inventory cost of distributor is high compared to retailer and manufacturer.

As a result of the above observation, to maximize the profit of the logistics companies need to focus to reduce the average inventory cost of distributor in introduction and decline phase and also decrease the average inventory cost of retailer in growth and maturity phase of the PLC.

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Figure 5. Inventory at Manufacture, Retailer and Distributor at different stage of PLC

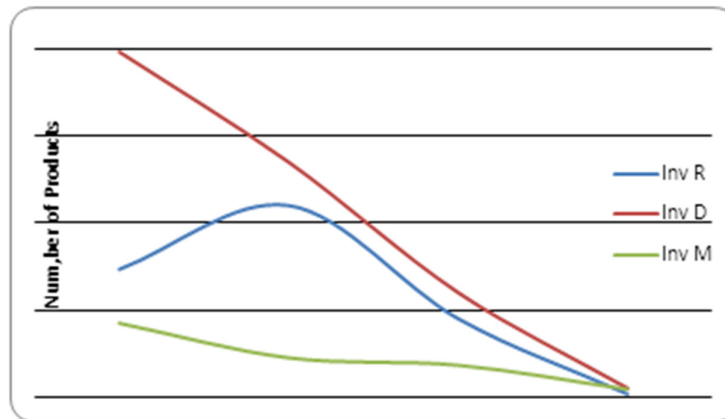
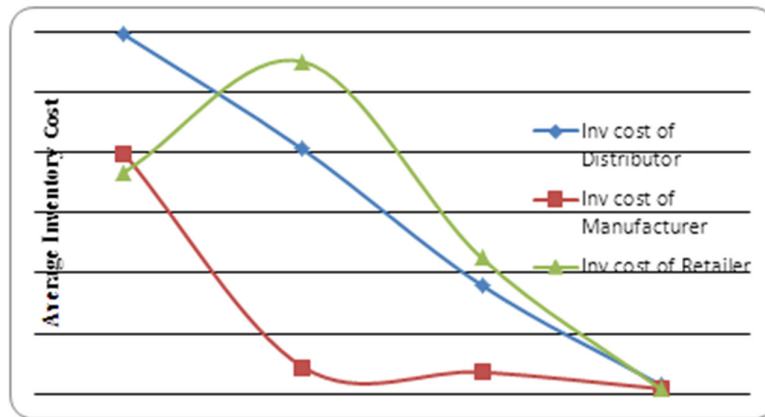
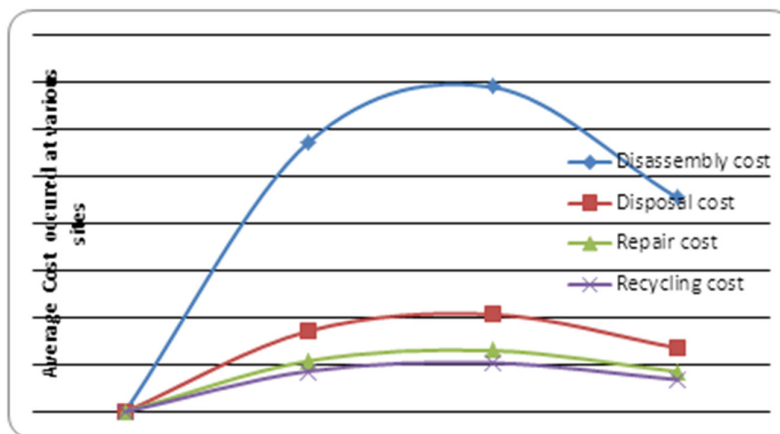


Figure 6. Inventory cost occurred at Manufacturer, Distributor and Retailer at various stage of PLC



The other cost parameters associated with the returns of products and parts are illustrated in the Figure.7, the average disassembly cost of parts, average disposal cost, average commercial return of the product and average recycling cost at various phases of PLC are compared. In this analysis, it is noted that the cost involved at disassembly sites are more when compared to the other cost. Industries should focus on how to control the disassembly cost of parts and the percentage of end of life returns(recycling cost) and end of use returns(part inventory) so that the disposal costs are reduced.

Figure 7. Disassembly, Disposal, Repair and Recycling cost incurred at various stage of PLC



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Appendix

| Indices | Parameters | Numerical Values |
|---------|--|------------------|
| S | Unit selling Price for the Product | 150 |
| a | Resource usage to produce one unit of Product | 1 |
| y | Unit direct manufacturing cost of product | 30 |
| e | Resource usage to repair one unit of Product | 1 |
| C | Max Capacity of repair site for Product | 9000 |
| d | Unit repair cost of Product | 1 |
| f | Set-up cost of disassembly site for Product | 5 |
| g | Set-up cost of repair site for Product | 5 |
| B | Max capacity of disassembly site to disassemble | 9000 |
| h | Unit disassembly cost for Part | 4 |
| m | Unit disposal cost for Part | 3 |
| r | Resource usage to disassemble one unit of Part | 1 |
| q | Unit requirements for Part to produce one unit of Product | 2 |
| n | Unit recycling cost for part in recycling site | 2 |
| o | Set-up cost of recycling site for part | 4 |
| s | Resource usage to recycle one unit of Part in recycling site | 1 |
| O | Max Capacity of Recycling site to recycle part | 9000 |
| HR | Unit Inventory holding cost at retailer of product | 5 |
| HD | Unit Inventory holding cost at Distributor of Product | 3 |
| HM | Unit Inventory holding cost at Manufacturer of Product | 2 |
| SH | Unit shipment cost of product | 3 |
| I | Inventory of a product at Manufacturing site at period $t=1$ | 1000 |
| F_1 | Inventory of a product at retailer at the end period t | 100 |
| F_2 | Inventory of a Product at distributor at the end period t | 200 |
| F_3 | Inventory of a Product at Manufacturer at the end period t | 300 |
| MD | Max capacity of truck to travel from Manufacturer to Distributor | 1000 |
| DR | Max Capacity of truck to travel from Distributor to Retailer | 2500 |
| z | Max percent of commercial returns | 0.6 |
| M | A big number | 10000 |
| M_1 | Max Percent of end-of-use returns | 0.3 |
| M_2 | Max Percent of end-of-life returns | 0.3 |
| L | Max number of recycling sites | 1 |
| A | Max capacity of the manufacturer plant | 250000 |

| | Introduction | | | Growth | | | | | Maturity | | | | | Decline | | |
|-------|--------------|------|------|--------|------|------|------|------|----------|------|------|------|------|---------|------|------|
| t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| D_t | 1500 | 1500 | 1500 | 2000 | 2500 | 3000 | 3500 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 3500 | 3000 | 2500 |
| N_t | 0.0 | 0.0 | 0.0 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.7 | 0.7 | 0.7 |

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V. Conclusions and Future Directions

In This paper analyzes the relationship between the demand and return of product in a CLSC during each phases of the product life cycle. The main contribution of the paper is to find the optimal inventory positioning of products and parts at different sites with different demand and return variations during four phases of product life cycle. The proposed mathematical model is solved by IBM ILOG CPLEX OPL studio. To analyse the performance of the model, a numerical example is considered with a linear function of demand and various percentage of returns during different phases of PLC.

The computational result of the decision variables at different sites of the CLSC for the given period length of 16 is analysed. Furthermore, to validate the results a sensitivity analysis is performed. The results of the analysis shows that the manufacturer can take appropriate decision at all the phases of PLC by considering the percent of return product obtained as commercial return at repair site and part inventory.

In addition, it is important for companies to focus on designing and implementing suitable policy interventions to ensure profitability of the supply chain. As one of the recommendations, we propose to devise suitable buy-back or revenue-sharing contract arrangements with distributor which would incentivise them suitably to hold more inventories in the earlier stages of product introduction. Similar arrangements can be considered for implementation in the retailer stages as well.

Further, since the disassembly stage is facing higher cost runs, it is recommended that this stage is properly integrated within the entire supply chain. Also, process improvement methodologies could be adopted to make this stage more efficient. Similar policies can be devised for managing the recycling and repair sites.

Several assumptions in this paper can be relaxed for future research. Lead time can be included at various sites of the CLSC and the return of product from customer to retailer, retailer to distributor, and distributor to manufacturer can be adopted in the model. Uncertainty in the demand rate and return rate also deserve further investigation when considering the product life cycle in CLSC.

Apart from this, we observed that the current model is computationally intensive even for small scale problems. For large scale implementations, the computational times are expected to be prohibitive. Thus, meta-heuristic algorithms like Very Large Scale Neighborhood Search, Genetic Algorithm and Particle Swarm optimization may be proposed for solving this problem

References

1. Che-Fu, Hsueh. 2011. "An inventory control model with consideration of remanufacturing and product life cycle." *International journal of Production Economics* 133: 645-652. doi:10.1016/j.ijpe.2011.05.007.
2. Jianmai, Shi, Guoqing Zhang, and Jichang Sha. 2011. "Optimal production planning for a multi-product closed loop system with uncertain demand and return." *Computers & Operations Research* 38: 641-650. doi:10.1016/j.cor.2010.08.008.
3. Kannan, G, P, Sasikumar, and K, Devika. 2010. "A genetic algorithm approach for solving a closed loop supply chain model: A case of Battery recycling." *Applied Mathematical Modeling* 34: 655-670. doi:10.1016/j.am.2009.06.021.
4. Kannan, G, A, Noorul Haq, and K, Devika. 2009. "Analysis of closed loop supply chain using genetic algorithm and Particle swarm optimisation." *International journal of production Research* 45: 1175-200. doi:10.1080/00207540701543585
5. Patroklos Georgiadis, Dimitrios Vlachos, and George Tagaras. 2006. "The Impact of Product Lifecycle on Capacity Planning of Closed-loop Chains with Remanufacturing." *Production and Operations Management Society* 15(4): 514-527. doi:1059-1478/06/1504/514.
6. Ronald, S and Tibben-Lembke. 2002. "Life after death: reverse logistics and the Product life cycle," *International Journal of Physical Distribution and Logistics Management* 32: 223-244. doi:10.1108/09600030210426548.
7. Saman Hassanzadeh Amin, and Guoqing Zhang. 2012. "A proposed mathematical model for closed loop network configuration based on product life cycle." *International Journal of Advanced Manufacturing Technology* 58:791-801. doi:10.1007/s00170-011-3407-2.
8. Sasikumar, P, G, Kannan, and A, Noorul Haq. 2010. "A multi-echelon reverse logistics network design for product recovery- a case of truck tire remanufacturing." *International Journal of Advanced Manufacturing Technology* 49:1223-1234. doi:10.1007/s00170-009-2470-4.
9. Sebnem Ahiska, and Russell, E, King. 2010. "Life cycle inventory policy characterizations for a single- product recoverable system." *International Journal of Production Economics* 124: 51-61. doi:10.1016/j.ijpe.2009.08.033.
10. Subramanian, P, N, Ramkumar, T, T, Narendran, and K, Ganesh. 2013. "PRISM: Priority based simulated annealing for a closed loop supply chain network design problem." *Applied Soft Computing* 13:1121- 1135. doi: 10.1016 / j. asoc. 2012. 10. 004.

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