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On $\beta^* g^*$-Closed Sets in Topological Spaces

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Abstract: *The purpose of this paper is to define and study $\beta^* g^*$-closed sets and $\beta^* g^* p$-closed sets, $\beta^* g^* s$-closed sets in Topological spaces.*

Keywords: $\beta^* g^*$-closed set, $\beta^* g^* p$-closed set, $\beta^* g^* s$-closed set and $\beta^* g$-open set.

1. INTRODUCTION AND PRELIMINARIES

In 1970, Levine [6] first considered the conceot of generalized closed (briefly, g-closed) sets were defined and investigated. Arya and Nour [2] defined generalized semi open (briefly, gs-open) sets using semi open sets. Veerakumar [11], S. Yuksel and Becern [12], A. Acikgoz [1] introduced g^*- -closed set, β^*- sets and $\beta^* g$ - closed sets respectively. We introduced a new class of sets $\beta^* g^*$-closed sets and study their simple properties.

Throughout this paper $(X,\tau), (Y,\sigma)$ and (Z,η) $(or\ X, Y, Z)$ represents topological spaces on which no seperaxion axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$, $int(A)$ and A^c $(or\ X - A)$ denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition: 1.1 A subset A of a topological space (X,τ) is called:

- pre-open [8] $A \subseteq int(cl(A))$,
- semi open [5] $A \subseteq cl(int(A))$,

The family of all preopen sets (resp. semi open sets) in X will be denoted by $po(X)$ (resp. $so(X)$). A semi closure (resp. pre closure) of a subset A of X denoted by $scl(A)$ (resp. $pcl(A)$) is defined to be the intersection of all semi closed (resp. pre closed) sets containing A. A semi interior (resp. pre interior) of a subset X denoted by $s\ int(A)$ (resp. $p\ int(A)$ is defined to the union of all semi open (resp. pre open) sets contained in A.

Definition: 1.2 A subset A of a topological space (X,τ) is called:

- a generalized closed set (briefly g-closed) [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)
- a - closed [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open set in (X,τ)

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- a -closed [7] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ)
- a -closed[2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ)

The complements of above sets are called their respective open sets.

Definition: 1.3 A subset A of a space is called a -set [12] if where U is open and

Definition: 1.4 A subset A of a space is called a -closed set [1] if whenever and U is -set in X.

2. -Closed Set

Definition: 2.1. A subset A of a space is called -closed set if $\subseteq U$ whenever $\subseteq U$ and U is a -open in X.

Definition: 2.2. A subset A of a space is called -closed set if $\subseteq U$ whenever $\subseteq U$ and U is a -open in X.

Definition: 2.3. A subset A of a space is called -closed set if $\subseteq U$ whenever $\subseteq U$ and U is a -open in X.

Theorem: 2.4. Let be a topological space. Then we have

- Every closed set is a -closed set.
- Every -closed set is a -closed set.

Proof: (i) Let A be a closed set in and U be a -open set such that $\subseteq U$. Since A is closed, cl (A) = A, So $\subseteq U$. Hence A is -closed set in

(ii) Let A be a -closed set in and $\subseteq U$ where U is -open set. Since every open set is a -open set, So U is an open set of Since A is a -closed set, we obtain that $\subseteq U$ hence A is a g-closed set of

Remark: 2.5. The converse of the above theorem need not be true as seen from the following examples.

Example: 2.6. Let and . Then the subset is a -closed set, but it is not a closed set.

Example: 2.7. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{c\}, X\}$. Then subset $A = \{a\}$ is a g-closed set, but it is not a -- closed set.

Theorem: 2.8. Let (X , τ) be a topological space. Then we have

- Every - closed set is a - closed set
- Every - closed set is a - closed set

Proof: (i) Assume that A is a β^* g*- closed set in (X , τ) and $A \subseteq U$ where U is a β^* g- open set. We have $pcl(A) \subseteq cl(A) \subseteq U$. Therefore $pcl(A) \subseteq U$. Hence A is a β^* g* p - closed set in (X , τ)

(ii) Assume that A is a -closed set in (X , τ) and $A \subseteq U$ where U is a - open set. We have $scl(A) \subseteq cl(A) \subseteq U$. Therefore $scl(A) \subseteq U$. Hence A is a β^* g* s - closed set in (X , τ).

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Example: 2.9. Let $X = \{a, b, c, d\}$ and$\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{b\}$ is a $\beta^* g^* p$-closed set, but it is not a $\beta^* g^*$-closed set.

Example: 2.10. Let $X = \{a, b, c, d\}$ and$\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{b, c\}$ is a $\beta^* g^* s$-closed set, but it is not a $\beta^* g^*$-closed set.

Theorem: 2.11. Let (X, τ) be a topological space. Then we have

- Every $\beta^* g^* p$- closed set is a gp-closed set.
- Every $\beta^* g^* s$-closed set is a gs-closed set.

Proof: (i) Assume that A is a $\beta^* g^* p$ - closed set of (X , τ). Let $A \subseteq U$ where U is a β^* g- open set. Since every open set is a $\beta^* g^*$-open set. Since A is a $\beta^* g^* p$ -closed set, Therefore $pcl(A) \subseteq U$. Hence A is a gp-closed set of (X, τ).

(ii) Assume that A is a $\beta^* g^* s$ - closed set of (X , τ). Let $A \subseteq U$ where U is a $\beta^* g^*$- open set. Since every open set is a $\beta^* g^* -$open set. Since A is a $\beta^* g^* s$ -closed set, Therefore $scl(A) \subseteq U$. Hence A is a gs-closed set of (X, τ).

Remark: 2.12. The converse of the above theorem need not be true as seen from the following examples.

Example: 2.13. Let $X = \{a, b, c, d\}$ and$\tau = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, X\}$. Then the subset $A = \{b, d\}$ is a gp-closed set, but it is not a $\beta^* g^* p$ -closed set.

Example: 2.14. Let $X = \{a, b, c, d\}$ and$\tau = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\} X\}$. Then the subset $A = \{c\}$ is a gs-closed set, but it is not a $\beta^* g^* s$ -closed set.

Theorem: 2.15. Let (X , τ) be a topological space. Then we have

- Every $\beta^* g^* -$closed set is a gp-closed set
- Every $\beta^* g^* -$closed set is a gs-closed set

Proof: (i) Assume that A is a $\beta^* g^*$- closed set of (X , τ). Let $A \subseteq U$ where U is a $\beta^* g^*$- open set. Since every open set is a $\beta^* g^*$–open, we have $pcl(A) \subseteq U$. Hence A is a gp -closed set of (X , τ).

(ii) Assume that A is a $\beta^* g^* -$closed set of (X , τ). Let $A \subseteq U$ where U is a $\beta^* g^*$- open set. Since every open set is a $\beta^* g^*$–open, we have $scl(A) \subseteq U$. Hence A is a gs -closed set of (X , τ).

Remark: 2.16. The converse of the above theorem need not be true as seen from the following examples.

Example: 2.17. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b\}, \{b, c, d\}, X\}$. Then the subset $A = \{d\}$ is a gp -closed set, but it is not a $\beta^* g^*$-closed set.

Example: 2.18. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b\}, \{b, c, d\}, X\}$. Then the subset $A = \{c\}$ is a gs -closed set, but it is not a $\beta^* g^*$-closed set.

Remark: 2.19. A β^* - Set is independent from $\beta^* g^*$-closed set as it can be seen from the next two examples.

Example: 2.20. Let $X = \{a, b, c, d\}$ and$\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{a\}$ is a β^*-set, but it is not a $\beta^* g^*$ -closed set.

Example: 2.21. Let $X = \{a, b, c, d\}$ and$\tau = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$. Then the subset $A = \{a, b, d\}$ is a $\beta^* g^*$-closed set, but it is not a β^* -set.

Theorem: 2.22. If A and B are $\beta^* g^*$-closed, then $A \cup B$ is a $\beta^* g^*$-closed set.

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Proof: Let A and B are $\beta^* g^*$-closed sets in X. Let U be $\beta^* g$-open set in X such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\beta^* g$-closed sets. $cl(\mathrm{A}) \subseteq U$ and $cl(\mathrm{B}) \subseteq U$. Hence $cl(A \cup B) = cl(A) \cup cl(B) \subseteq U$. Therefore $A \cup B$ is $\beta^* g^*$-closed set whenever A and B are $\beta^* g^*$-closed set.

Remark: 2.23. The finite intersection of two $\beta^* g^*$-closed sets need not be $\beta^* g^*$-closed set.

Example: 2.24. Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then the subset $A = \{a, b, c\}$ and $\{a, b, d\}$ are $\beta^* g^*$-closed sets, but $\{a, b, c\} \cap \{a, b, d\} = \{a, b\}$ is not a $\beta^* g^*$-closed set.

Theorem: 2.25. If $A \subseteq B \subseteq cl(A)$ and A is a $\beta^* g^*$-closed subset of (X , τ), then B is also a $\beta^* g^*$-closed subset of (X , τ).

Proof: Let U be a $\beta^* g$-open subset, such that $A \subseteq B \subseteq U$, Since A is $\beta^* g^*$-closed subset of (X , τ). $cl(A) \subseteq U$, by hypothesis $A \subseteq B \subseteq cl(A)$, $cl(\mathrm{A}) = \mathrm{cl}(\mathrm{B})$. Hence $cl(\mathrm{B}) \subseteq \mathrm{U}$ whenever $\mathrm{B} \subseteq \mathrm{U}$, Therefore B is $\beta^* g^*$-closed subset of (X , τ).

Theorem: 2.26. For any topological space (X , τ), every singleton $\{x\}$ of X is a $\beta^* g$-open set.

Proof: Let $x \in X$. Let $\{x\} \in \tau$, then $\{x\}$ is a $\beta^* g$-open set. If $\{x\} \notin \tau$, then $int(\{x\}) = \emptyset = cl\big(int(\{x\})\big)$, so $\{x\}$ is a $\beta^* g$-open set.

Theorem: 2.27. A subset A of X is $\beta^* g^*$-closed set in X if and only if $cl(\mathrm{A}) - \mathrm{A}$ Contains no nonempty $\beta^* g$-closed set in X.

Proof: Suppose that F is a nonempty $\beta^* g$-closed subset of $cl(\mathrm{A}) - \mathrm{A}$. Now $F \subseteq cl(\mathrm{A}) - \mathrm{A}$. $F \subseteq cl(\mathrm{A}) \cap \mathrm{A}^c$. Therefore $F \subseteq cl(\mathrm{A})$ and $F \subseteq \mathrm{A}^c$. Since F^c is $\beta^* g$-open such that $A \subseteq F^c$ and A is $\beta^* g^*$-closed, $cl(A) \subseteq F^c$, ie $F \subseteq cl(A)^c$. Hence $F \subseteq cl(A) \cap [\mathrm{cl}(\mathrm{A})]^c = \phi$. Ie, $F = \phi$. Thus $cl(\mathrm{A}) - \mathrm{A}$ contains no nonempty $\beta^* g^*$-closed set.

Conversely, Assume that $cl(\mathrm{A}) - \mathrm{A}$ contains no nonempty $\beta^* g$-closed set. Let $A \subseteq U$, U is $\beta^* g$-open. Suppose that $cl(\mathrm{A})$ is not contained in U. Then $cl(\mathrm{A}) \cap \mathrm{U}^c$ is a nonempty $\beta^* g$-closed set and contained $cl(\mathrm{A}) - \mathrm{A}$ which is contradiction. Therefore $cl(\mathrm{A}) \subseteq \mathrm{U}$ and hence A is $\beta^* g$-closed set.

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