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ON r δ-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: *A new set called regular δ-closed (briefly rδ) sets is introduced in this research which arises between the class of δ-closed sets and the class of all regular g-closed sets. In addition we study some of its vital properties and examine the relations between the associated topology.*

Keywords: Topological spaces, closed sets, separation axioms, δ-closed sets, generalized closed sets, regular generalized closed sets

1. INTRODUCTION AND PRELIMINARIES

Norman Levine introduced and studied generalized closed (briefly g-closed) sets [11] and semi-open sets [12] in 1963 and 1970 respectively. Arya and Nour [3] defined generalized semi-closed (briefly gs-closed) sets for obtaining some characterizations of s-normal spaces in 1990.

Njåstad [17] introduced the concepts of α-sets (known as α-open sets) and β-Sets (known as β-open sets) for topological spaces. Andrijević [1] called β-sets as semi-preopen sets. H. Maki called generalized α-open sets in two ways and introduced generalized α-closed(briefly gα-closed) sets[13] and α-generalized closed(briefly αg-closed) sets[14] in 1993 and 1994 respectively. Dontchev [6] introduced generalized semi-preclosed (briefly gsp-closed) sets in 1995. Palaniappan and Rao [18] introduced regular generalized closed (briefly rg-closed) sets in 1993. Gnanambal [10] introduced generalized pre regular closed (briefly gpr-closed) sets. In this paper, we study the relationships of δ-closed sets with regular generalized closed sets.We obtain basic properties of regular δ -closed sets.

Throughout this paper $(X,\tau),(Y,\sigma)$ and (Z,η) $(or\ X,Y,Z)$ represents topological spaces on which no seperaxion axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), $cl(A)$, $int(A)$ and A^c $(or\ X - A)$ denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition:1.1 A subset A of a topological space (X,τ) is called:

1. pre open [16] $A \subseteq int(cl(A))$,
2. semi open[12] $A \subseteq cl(int(A))$,
3. regular open [19] $A = int(cl(A))$.

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Definition:1.2 A subset A of a topological space (X, τ) is called:

1. a generalized closed set (briefly g-closed) [11] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ),
2. a αg-closed [13] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ),
3. a $\hat{g}$ –closed [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open set in (X, τ).
4. a gs –closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).

The complements of above sets are called their respective open sets.

Definition 1.3 A subset A of a topological space (X,τ) is called Regular δ-closed (rδ-closed) if A = $cl_\delta(A)$ where

$cl_\delta(A) = \{x \epsilon X: int(cl_\delta(U)) \cap A \neq \emptyset, U \in \tau \; and \; x \; \epsilon U\}$ whenever $A \subseteq U$ and U is regular open in (X,τ).

2. R δ - closed sets

Theorem : 2.1 Every rδ- closed set is a g- closed set.

Proof: Obvious.

The converse of the above theorem is not true in general as it can be seen from the following example.

Example: 2.2 Let X = {x, y, z} = τ = {∅, X, {x}, {y, z}} and D = {y}. D is not a g- closed set since {y} is a g-open set of (X,τ) such that $D \subseteq \{y\}$ but $cl_\delta(D) = cl_\delta\{\{y\}\} = \{y, z\} \subseteq \{y\}$

The following theorem shows that the class of rg-closed sets is properly contained in the class of αg-closed sets, the class of gs-closed sets, the class of gsp-closed sets, the class of gp-closed sets, the class of gpr-closed sets, the class of αg-closed.

Corollary: 2.3 Union (intersection) of any rδ -closed sets is again rδ -closed.

Corollary: 2.4 Let A be a rδ -closed of (X, τ). Then A is closed if and only if cl(A)-A is semi-closed.

Corollary:2.5 In a submaximal space (X, τ), every rδ -closed set is closed.

Theorem :2.6 Let A be a rδ -closed set of (X, τ). Then cl(A)-A does not contain any non-empty semi-closed set.

Proof: Let F be a semi-closed subset of (X, τ) such that F ⊆ cl(A)-A. Then F ⊆ X-A. This implies A ⊆ X-F. Now X-F is semi-open set of (X, τ) such that A ⊆ X-F. Since A is a rδ -closed set of (X, τ), then cl(A) ⊆ X-F. Thus F ⊆ X-cl(A). Now F ⊆ cl(A) ∩ (X-cl(A)) =∅.Therefore F = ∅.

Theorem :2.7 Every rδ-closed set is rg-closed set but not conversely.

Proof: Let A be rδ -closed set of (X, τ). Let G be a regular open set such that A ⊆ G. Then A ⊆ int(cl(G)). Since int(cl(G)) is semi-open set containing the rδ -closed set, then cl(A) ⊆ int(cl(G)). Therefore A is an rg-closed set.

Theorem :2.8 Every closed (resp. $\hat{g}$-closed) set is an r δ-closed set.

Proof: From the following example we will prove that the converse of the above theorem is not true.

Example :2.9 Let X = {a, b, c} and τ = {∅, X, {a}, {b, c}}. Consider A = {b}. A is not a closed set. However, A is an rδ-closed set.

Example :2.10 Let X = {a, b, c} and τ = {∅, X, {a}, {a, c}}. Consider A = {a, b}. A is not an rδ -closed set. However, A is a g-closed set.

Therefore, the class of rδ-closed sets is properly contained in the class of g-closed sets and properly contains the class of closed sets.

Theorem :2.11 Every rδ-closed set is αg-closed, gα-closed, and gs-closed set but not conversely.

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Proof: Follows from the previous theorem and the fact that every rδ-closed set is an αg-closed set, gs-closed set and scl(A) ⊆ αcl(A) ⊆ cl(A) for any subset A of a space (X, τ).

Consider the space (X, τ) in the Example. The set B = {c} is αg-closed and gα-closed and hence sg-closed and gs-closed. But B is not an rδ -closed set.

Thus the class of rδ-closed sets properly contains the class of αg-closed sets, the class of gα-closed sets, the class of gs-closed sets and the class of sg-closed sets. Next we show that this new class also properly contains the class of rg-closed sets, the class of gpr-closed sets and the class of gsp-closed sets.

Theorem :2.12 Let A be a rδ- closed set of a topological space (X,τ), Then, $pcl_{\delta}(A)$is rδ- closed.

Proof:A subset A of (X,τ), $scl_{\delta}(A) = A \cup int(cl_{\delta}(A))$and $pcl_{\delta}(A) = A \cup int(cl_{\delta}(A))$.

Since $pcl_{\delta}(A)$ is the union of two rδ- closed sets A and $cl_{\delta}(int(A))$.

Example:2.13 Let X = {a, b, c} τ = {∅,X,{a},{b},{a, b} {a, c}}. Consider A = {c}. Here A is not regular open. This A is δ-closed and$scl_{\delta}(A) = pint(A) = \emptyset$ is rδ- closed.

Theorem : 2.14 If A is a rδ- closed set of (X), such that. $A \subseteq B \subseteq cl_{\delta}(A)$, then B is also rδ- closed set of (X,τ).

Proof: Let U be a regular open set of (X,τ) such that $B \subseteq U$ Then $A \subseteq U$ since A is δ- closed the $cl_{\delta}(A) \subseteq U$.

$cl_{\delta}(B) \subseteq cl_{\delta}(cl(A)) = cl_{\delta}(A) \subseteq U$. B is rδ- closed set of (X,τ). $B \subseteq U$

Theorem :2.15 Let A be a locally closed set of (X, τ). Then A is rδ -closed if and only if A is closed.

Proof: Obvious.

Theorem :2.16 If A is regular open, then sint(A) is rδ -closed.

Proof: First we note that for a subset A of (X, τ), scl(A) = A ∪ int(cl(A)) and pcl(A) = A ∪ cl(int(A)). Moreover, sint(A) = A ∩ cl(int(A)) and pint(A) = A ∩ int(cl(A)).

(1) Since cl(int(A)) is a closed set, then A and cl(int(A)) are rδ -closed sets. By the Theorem 3, A ∩ cl(int(A)) is also a rδ -closed set.

Theorem :2.17 If A is regular open ,then pcl(A) is rδ -closed.

Proof: pcl(A) is the union of two rδ -closed sets A and cl(int(A)). Again by the Theorem 3,

pcl(A) is rδ -closed.

Theorem :2.18 If A is regular open, then pint(A) and scl(A) are also rδ -closed sets.

Proof: Since A is regular open, then A = int(cl(A)). Then scl(A) = A ∪ int(cl(A)) = A. Thus scl(A) is rδ -closed. Similarly pintA is also an rδ -closed set.

Theorem :2.19 A is a rδ- closed of (X,τ) such that if and only if $cl_{\delta}(A)$ -A does not contain any non-empty δ- closed set.

Proof: Let U be a δ –regular open set of (X, τ) such that A ⊆U. If $cl_{\delta}(A) \subseteq U$, then $cl_{\delta}(A) \cap C(U) = \emptyset$. Since $cl_{\delta}(A)$ is a closed set, then by [12], $\emptyset = cl_{\delta}(A) \cap C(U)$ is a δ -closed set of (X, τ). Then $\emptyset = cl(A) \cap C(U) \subseteq cl_{\delta}(A)$-A. So $cl_{\delta}(A)$-A contains a non-empty δ -closed set. A contradiction. Therefore A is r δ -closed.

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