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STRONGLY g*-CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract: *The purpose of this paper is to define and study sg^*-closed sets in bitopological spaces*

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1. INTRODUCTION AND PRELIMINARIES

A triple $(X, \tau_{1,}\tau_2)$ where X is a non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly [3] initiated the study of such spaces.

Throughout this paper (X, τ_1, τ_2) or simply X represents the bitopological spaces on which no seperaxion axioms are assumed unless otherwise mentioned. For any subset $A \subseteq X$, τ_i-$int(A)$ and , τ_i-$cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i, respectively.

Definition 1.1: A subset A of a topological space (X, τ) is said to be

(1) Semi-open [4] if $A \subseteq cl(int(A))$

(2) Regular open if $A = int(cl(A))$

Definition 1.2: A subset A of a topological space (X, τ) is said to be generalized closed **(briefly g-closed)** [1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 1.3: A subset A of a topological space (X, τ) is said to be generalized* closed **(briefly g^*-closed)** [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

Definition 1.4: A subset A of a bitopological space (X, τ_1, τ_2) is said to be

(1) $\tau_1\tau_1$-Semi-open [2] if $A \subseteq \tau_2 - cl(\tau_1 - int(A))$

(2) $\tau_1\tau_2$ -Regular open if $A = int(cl(A))$

Definition 1.5: A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-g- closed) [ ] if $\tau_2 cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open.

Definition 1.6: A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-generalized* closed ($\tau_1\tau_2$-g^*- closed) [ ] if $\tau_2 - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-g open.

2. STRONGLY g*-CLOSED SETS

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Definition 2.1: Let (X, τ_1, τ_2) be a bitopological space and A be its subset, then A is a strongly g^*-closed set (briefly sg^*-closed) if τ_2-$cl(\tau_1 - int(A)) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- g open

Theorem 2.2: Let (X, τ_1, τ_2) be a bitopological space. Every closed set is strongly g^*-closed set, but not conversely.

Proof: Suppose that A is closed. Let U be an open set containing A. Then τ_2-$cl(\tau_1 - int(A)) \subseteq \tau_2 - cl(A) = A$, which implies, τ_2-$cl(\tau_1 - int(A)) \subseteq U$. Hence, A is a strongly g^*-closed set.

Example 2.3: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then the set $\{a, c\}$ is a strongly g^*-closed set but not a closed set.

Theorem 2.4: If a subset A of a bitopological space (X, τ_1, τ_2) is g^*-closed then it is strongly g^*-closed in X, but not conversely.

Proof: Suppose A is g^*-closed in (X, τ_1, τ_2). Let G be an open set containing A in X. Then G contains $\tau_2 - cl(A)$ and $G \supseteq \tau_2 - cl(A) \supseteq \tau_2 - cl(\tau_1 - int(A))$. Thus, A is strongly g^*-closed in X.

Example 2.5: Let $X = \{a, b, c\}$ with topologies $\tau_1 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $\tau_2 = \{\emptyset, \{a, c\}, X\}$. In this topological space the subset $\{a\}$ is strongly g^*-closed but not a g^*-closed set.

Theorem 2.6: If a subset A of a topological space (X, τ_1, τ_2) is both open and strongly g^*-closed, then it is closed.

Proof: Suppose a subset A of X is both open and strongly g^*-closed. Then $A \supseteq \tau_2 - cl(\tau_1 - int(A)) \supseteq \tau_2 - cl(A)$ and so $A \supseteq \tau_2 - cl(A)$. Since $\tau_2 - cl(A) \supseteq A$, we have, $A = \tau_2 - cl(A)$. Thus A is closed in X.

Theorem 2.7: If a subset A of a bitopological space (X, τ_1, τ_2) is both strongly g^*-closed and semi-open then it is g^*-closed.

Proof: Suppose A is both strongly g^*-closed and semi-open in X, let G be an open set containing A. As A is strongly g^*-closed, $G \supseteq \tau_2 - cl(\tau_1 - int(A))$. Now, $G \supseteq \tau_2 - cl(A)$, since A is semi-open. Thus A is g^*-closed in X.

Corollary 2.8: If a subset A of a bitopological space (X, τ_1, τ_2) is both strongly g^*-closed and open then it is a g^*-closed set.

Proof: Suppose A is both strongly g^*-closed and open in X, let G be an open set containing A.

As A is strongly g^*-closed, $G \supseteq \tau_2 - cl(\tau_1 - int(A))$ and $G \supseteq \tau_2 - cl(A)$, since A is open. Thus, A is g^*-closed in X.

Theorem 2.9: A subset A is strongly g^*-closed if and only if $\tau_2 - cl(\tau_1 - int(A)) - A$ contains no non-empty closed set.

Necessity: Suppose that F is a non-empty closed subset of $\tau_2 - cl(\tau_1 - int(A)) - A$. i.e., $F \subseteq \tau_2 - cl(\tau_1 - int(A)) \cap (X - A)$. Then $F \subseteq \tau_2 - cl(\tau_1 - int(A))$ and $F \subseteq (X - A)$. Since $X - F$ is an open set and A is strongly g^*-closed, $\tau_2 - cl(\tau_1 - int(A)) \subseteq (X - F)$. i.e., $F \subseteq (X - \tau_2 - cl(\tau_1 - int(A)))$. Hence, $F \subseteq \tau_2 - cl(\tau_1 - int(A)) \cap (X - (\tau_2 - cl(\tau_1 - int(A))) = \emptyset$. i.e., $F = \emptyset$. Thus, $\tau_2 - cl(\tau_1 - int(A)) - A$ contains no non-empty closed set.

Sufficiency: Conversely, assume that $\tau_2 - cl(\tau_1 - int(A)) - A$ contains no non-empty closed set. Let $A \subseteq U, U$ is g-open. Suppose that $\tau_2 - cl(\tau_1 - int(A))$ is not contained in U. Then $\tau_2 - cl(\tau_1 - int(A)) \cap (X - U)$ is a non-empty closed set and contained in $\tau_2 - cl(\tau_1 - int(A)) - A$ which is a contradiction. Therefore, $\tau_2 - cl(\tau_1 - int(A)) \subseteq U$ and hence A is strongly g^* - closed.

Corollary 2.10: A strongly g^* - closed set A is regular closed if and only if $\tau_2 - cl(\tau_1 - int(A)) - A$ is closed and $\tau_2 - cl(\tau_1 - int(A)) \supseteq A$.

Proof: Assume that A is regular closed. Since $\tau_2 - cl(\tau_1 - int(A)) = A$, $\tau_2 - cl(\tau_1 - int(A)) - A = \emptyset$ is regular closed and hence closed.

Conversely, assume that $\tau_2 - cl(\tau_1 - int(A)) - A$ is closed. By Theorem 9, $\tau_2 - cl(\tau_1 - int(A)) - A$ contains no non-empty closed set. Therefore, $\tau_2 - cl(\tau_1 - int(A)) - A = \emptyset$. Thus, A is regular closed.

Theorem 2.11: Suppose that $B \subseteq A \subseteq X$, B is a strongly g^*- closed set relative to A and that both open and strongly g^*-closed subset of (X, τ_1, τ_2) then B is a strongly g^* - closed set relative to (X, τ_1, τ_2).

Proof : Let $B \subseteq G$ and G be an open set in (X, τ_1, τ_2). But given that $B \subseteq A \subseteq X$, therefore $B \subseteq A$ and $B \subseteq G$. This implies, $B \subseteq A \cap G$. Since B is strongly g^* - closed relative to A, $\tau_2 - cl(\tau_1 - int(B)) \subseteq A \cap G$. i.e., $A \cap \tau_2 - cl(\tau_1 - int(B)) \subseteq A \cap G$. This implies, $A \cap \tau_2 - cl(\tau_1 - int(B)) \subseteq G$. Thus, $A \cap \tau_2 - cl(\tau_1 - int(B)) \cup (X - (\tau_2 - cl(\tau_1 - int(B)))) \subseteq G \cup (X - \tau_2 - cl(\tau_1 - int(B)))$. Also, $B \subseteq A$ which implies $\tau_2 - cl(\tau_1 - int(B)) \subseteq \tau_2 - cl(\tau_1 - int(A))$

Corollary 2.12: Let A be strongly g^*-closed and suppose that F is closed then $A \cap F$ is a strongly g^*-closed set.

Proof: To show that $A \cap F$ is strongly g^*-closed, we have to show $\tau_2 - cl(\tau_1 - int(A \cap F)) \subseteq G$ whenever $A \cap F \subseteq G$ and G is g-open. Since $A \cap F$ is closed in A, we have $A \cap F$ is strongly g^*-closed in A. By Theorem 4.11, $A \cap F$ is strongly g^*-closed in (X, τ_1, τ_2), since $A \cap F \subseteq A \subseteq (X, \tau_1, \tau_2)$.

Theorem 2.13: If A is strongly g^*-closed and $A \subseteq B \subseteq \tau_2 - cl(\tau_1 - int(A))$, then B is strongly g^*-closed.

Proof: Given that $B \subseteq \tau_2 - cl(\tau_1 - int(A))$ then $\tau_2 - cl(\tau_1 - int(B)) \subseteq \tau_2 - cl(\tau_1 - int(A))$, $\tau_2 - cl(\tau_1 - int(A)) - B \subseteq \tau_2 - cl(\tau_1 - int(A)) - A$. Since $A \subseteq B$, and A is strongly g^*-closed, by Theorem 4.9, $\tau_2 - cl(\tau_1 - int(A)) - A$ contains no non-empty closed set and $\tau_2 - cl(\tau_1 - int(B)) - B$ contains no non-empty closed set. Again by Theorem 4.9, B is a strongly g^*-closed set.

Theorem 2.14: Let X and Y are bitopological spaces and let $A \subseteq Y \subseteq X$ and suppose that A is strongly g^*- closed in X then A is strongly g^*- closed relative to Y.

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Proof: Given that $A \subseteq Y \subseteq X$ and A is strongly g^*-closed in X. To show that A is strongly g^*-closed relative to Y, let $A \subseteq Y \cap G$, where G is g-open in X. Since A is strongly g^*-closed in X, $A \subseteq G$ implies $\tau_2 - cl(\tau_1 - int(A)) \subseteq G$. i.e., $A \subseteq Y \cap G$, where $Y \cap \tau_2 - cl(\tau_1 - int(A)) \subseteq Y \cap G$, where $Y \cap \tau_2 - cl(\tau_1 - int(A))$ is the closure of interior of A in Y. Thus A is strongly g^*-closed relative to Y.

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