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Colored Noise Reduction By Kalman Filter

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Abstract: *This paper deals with reduction of colored noise from audio signal to improve the audibility of the signal.In this design an audio signal is used as an input signal whose Auto regressive coefficients is calculated by AR-Yule method and is given to the system in a matrix form. Brown colored is used as a noisy signal. In this work Kalman filter is used to reduce colored noise from the audio signal.Kalman filter has the advantage of zero Gaussian noise. It works in a two-step algorithm. Prediction and estimation are the essential steps of it Aim of these designs is to reduce the noise from the signal. This system is designed and simulated using Matlab Simulink version R2013a.*

Keywords: Kalman filter, colored noise,auto regressive co efficients.

I. INTRODUCTION

The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) solution of the least-squares method. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. It is shown that the Kalman filter is a linear, discrete time, finite dimensional time-varying system that evaluates the state estimate that minimizes the mean-square error. The Kalman filter dynamics results from the consecutive cycles of prediction and filtering. The dynamics of these cycles is derived and interpreted in the framework of Gaussian probability density functions. Under additional conditions on the system dynamics, the Kalman filter dynamics converges to a steady-state filter and the steady-state gain is derived. The innovation process associated with the filter, that represents the novel information conveyed to the state estimate by the last system measurement, is introduced. The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state.

The Kalman filter addresses the general problem of trying to estimate the state of a $x€ R^n$ discrete-time controlled process that is governed by the linear stochastic difference equation
$x_{k+1} = A_k x_k + Bu_k + w_k$ ...... eq(1)
with a measurement $z € R^n$
$z_k = H_k x_k + v_k$ ....eq(2)
The random variables w_k and v_k represent the process and measurement noise respectively They are assumed to be independent (of each other), white, and with normal probability distributions.
P(w)=N(0,Q)
P(v)=N(0,R)
In practice, the *process noise covariance Q* and *measurement noise covariance R* matrices might change with each time step or measurement, however here we assume they are constant The n*n matrix A in the difference equation relates the state at the previous time step k-1

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to the state at the current step k , in the absence of either a driving function or process noise. Note that in practice A might change with each time step, but here we assume it is constant. The n*l matrix *B* relates the optional control input to the state *x*. The matrix in the measurement equation relates the state to the measurement *zk*. In practice H might change with each time step or measurement, but here we assume it is constant.

1.3 Computational Origins of The filter :

We define x_k^- € R^n to be the *priori* state estimate at step *k* given knowledge of the process prior to step *k*, and x_k € R^n to be our *a posteriori* state estimate at step *k* given measurement z_k . We can then define *a priori* and *a posteriori* estimate errors as

$$e_k^- \ \Xi\ x_k - x_k^- \quad \text{and} \quad e_k \ \Xi\ x_k - x_k \qquad \text{.........eq (3)}$$

The *a priori* estimate error covariance is then

$$P_k^- = E\,[e_k^-\, e_k^{-T}] \qquad \text{...........eq (4)}$$

And the a posteriori error covariance is

$$P_k^- = E\,[e_k^-\, e_k^{T}] \qquad \text{.............eq (5)}$$

In deriving the equations for the Kalman filter, we begin with the goal of finding an equation that computes an *a posteriori* state estimate x_k as a linear combination of an *a priori* estimate x^-_k and a weighted difference between an actual measurement z_k and a measurement prediction $H\,x^-_k$.

$$x_k = x_k^- + k\,(z_k - Hx_k^-) \qquad \text{...............eq(6)}$$

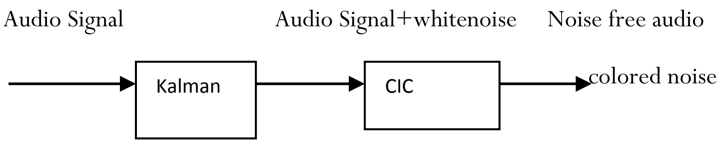
the difference $(z_k - Hx_k^-)$ is called measurement innovation or the residual. The residual reflects the discrepancy between the predicted measurement and the actual measurement z_k A residual of zero means that the two are in complete agreement.

The n*n matrix K is chosen to be the gain or blending factor that minimizes the *a posteriori* error covariance equation

$$K_k = P_k^-\, H^T\, (\,H\, P_k^-\, H^T + R\,)^{-1}$$

$$= (P_k^-\, H^T) \,/\, (\,H\, P_k^-\, H^T + R\,) \qquad \text{....... eq (7)}$$

as the measurement error covariance R approaches zero,the gain K weights the residual more heavily.On the other hand, as the *a priori* estimate error covariance P_k^- approaches zero, the gain*K* weights the residual less heavily as the measurement error covariance R approaches zero,the gain K weights the residual more heavily.



Noise suppression by kalman filter[3] : There are several noise reduction algorithms base on linear prediction have been proposed in case that noise signal is AWGN(additive white Gaussian noise).There are some advantages of using kalman filter.

Assuming that the speech signal d(n) is degraded by an additive observation noise v(n), a noisy speech signal r(n) is given by

r(n) = d(n)+v(n)

A noise suppression procedure consists in estimating the speech signal d(n) from the sole noisy speech signal r(n).Here it is assumed that the noise v(n) is additive noise with known variance σ_v^2.The noise variance may not be known in practice but it can be estimated in many methods. Our purpose of it is to achieve high performance noise suppression without sacrificing quality of the speech signal from the only noisy speech signal r(t) for the additive white and colored noise.

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The method utilizes the canonical state space models with a state equation considered of the speech signal, and an observation equation consisted of the speech signal and additive noise.The proposed algorithm is performed without the conception of the generated model of speech signal.The main tool in our algorithm is Kalman filter theory only.

For the $L_p * 1$ state vector x_p (n+1) of the proposed method

Define

$X_c(n+1)=d(n+1),d(n),\ldots\ldots\ldots\ldots\ldots,d(n-L_p+2)]^T$ ....eq(8)

Nothing the signal d(n) ,we give state equation :

State equation

$X_c (n+1) = \varphi_p x_p (n) + \delta_p (n+1)$ ....eq(9)

Where the $L_p * L_p$ transition matrix φ_p and the $L_p * 1$ driving source vector δ_p (n+1) are expressed as

$$\varphi_p = \begin{bmatrix} 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ldots & 0 \\ 0 \quad 1 \ldots\ldots\ldots\ldots\ldots & 0 \\ 0 \quad 0 \quad 1 \ldots\ldots\ldots\ldots\ldots & 0 \end{bmatrix}$$

$\delta_c (n+1) = [d(n+1),0,\ldots\ldots.0]^T$ ....eq(10)

furthermore define the $L_p * 1$ extended observation vector $y_p (n+1)=[1,r(n+1),\ldots\ldots\ldots.r(n-L_p+3)]^T$
the observation equation may be also expressed as
observation equation
$y_p (n+1) = M_p x_p (n+1) + \epsilon_p (n+1)$ ....eq(11)

where the $L_p * L_p$ observation matrix M_p and the $L_p * 1$ extended vector ϵ_p (n+1) are expressed as

$M_p = \varphi_p$,

$\epsilon_p (n+1) = [1,v(n+1),\ldots\ldots v(n - L_p + 3)]^T$ .....eq(12)

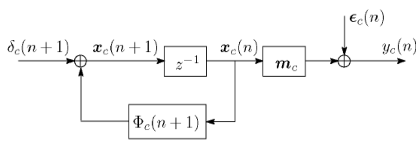

Fig: Block Diagram of the Overview of Noise

It is easily guessed that the proposed algorithm does not depends on both the estimation accuracy of the parameters of AR system and the order L_c of it, since the proposed algorithm does not include the conception of the generated model of signal,
CIC (Cascaded Integrated Comb Filter)[5]:

CIC filters were invented by Eugene B. Hogenauer, and are a class of FIR filters used in multi-rate digital signal processing. The CIC filter finds applications in interpolation and decimation. Unlike most FIR filters, it has a decimator or interpolator built into the architecture. The figure at the right shows the Hogenauer architecture for a CIC interpolator.

CIC filter is one of the simplest filter because the logic requirement of this filter is less as this filter does not require a multiplier. Since the filter co efficients are all unity multiplier is not required in that kind of filter. However the comb filter is not very efficient for removing noise in wide frequency range.For many applications the comb filter must be used in connection with one or more additional design. A cascaded Integrator Comb filter is a special class of linear phase, finite impulse response filter. For many applications which cannot tolerate this distortion.the comb filter must be used in conjunction with one or more additional digital filter

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stages.CIC filters generally used in multirate systems for better performance.The system is completely designed to reduce noise while noise is mixing continuously with the source signal.For developing such system a CIC filter is crude to eliminate noise from sine wave.

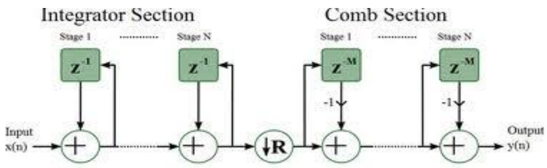


Fig: Block diagram of Cascaded Integrator Comb Filter

$$H(z) = [\textstyle\sum_{k=0}^{RM-1} Z^{-k}]^{N}$$
$$= [(1-Z^{-RM}) \,/\, (1-Z^{-1})]^{N}$$

Where R=Decimation or interpolation ratio

M=number of samples per stage

N=number of stages in filter

Characteristics of CIC filters are that it gives linear phase response and utilizes only delay and addition and subtraction.CIC filter structure The basic elements of a CIC filter are integrator filters and comb filters.

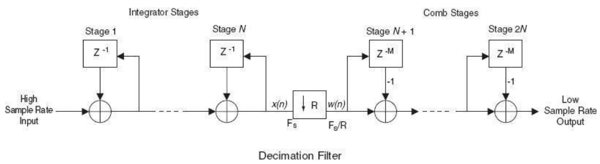
1.7Frequency characteristics :

The transfer function for a CIC filter is at f_s is

$H(z) = H_I(z)\, H_C(z) = = [(1-Z^{-RM}) \,/\, (1-Z^{-1})]^{N}$

This equation shows that even though a CIC has integrators in it, which by themselves have an infinite impulse responses , a CIC filter is equivalent to N FIR filters. Each having a rectangular impulse responses. Since all of the coefficients of these FIR filters are unity and therefore symmetric,a CIC filter also has a linear phase response and constant group delay.

$|H(f)| = |[(\sin \pi Mf)/(\sin \pi f/R)]^{N}|$



Block Diagram of CIC Filter

Here 10*10 matrix is being considered.By using AR-yule method the auto regressive co efficient of the input signal and the brown noise is calculated.

The matlab code for it is simulated to find out the AR coefficient.

$\delta_c\,(n+1) =$

$$\begin{pmatrix}
1.0 & 0.8237 & 0.1114 & -0.0471 & -0.0079 & 0.0151 & 0.0799 & -0.1224 & 0.0271 & 0.04231 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}$$

$\Phi_c(n+1) =$

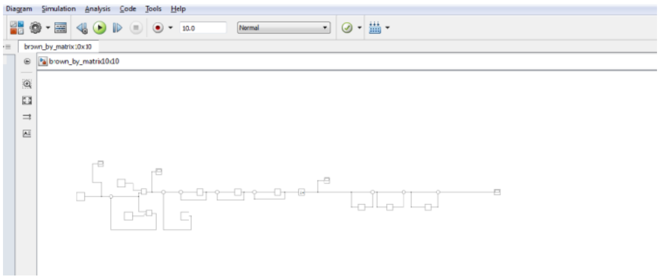
$$\begin{pmatrix}
1.0000 & 0.0943 & 0.9264 & -0.0745 & 0.0252 & -0.1387 & 0.0822 & 0.0252 & -0.0974 & 0.1850 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}$$

$X_c(n+1)=$

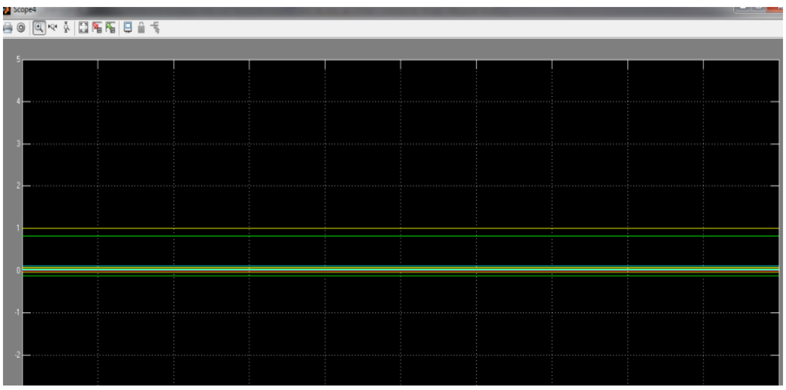
$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Results and Discussion :

These values are given and then simulated by Matlab R2013a.The circuit which is simulated after giving the AR values of audio sign. and brown noise is given below



After the simulation of the above circuit the scope block of the audio signal give the output like,



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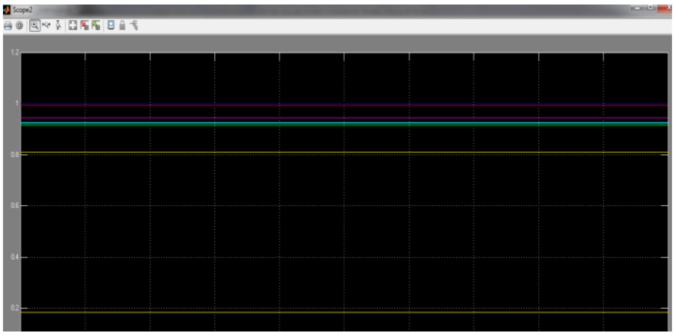
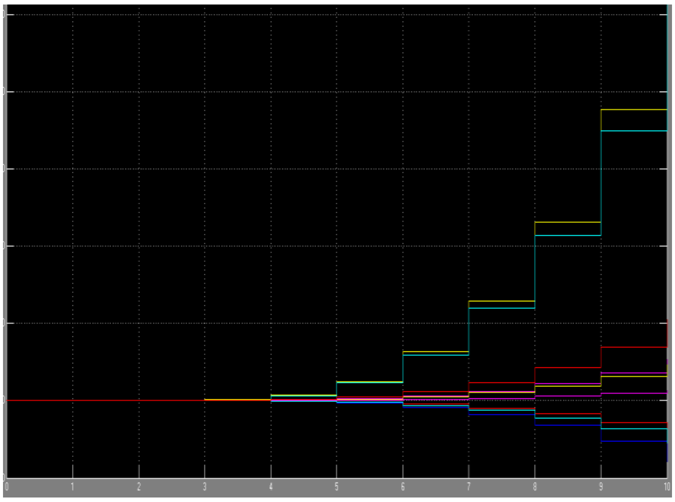
Output of the scope block after mixing brown noise with audio signal looks like,



Output of the final scope shown below where the noise is down to zero.The audio signal is amplified.



Conclusion :

In this paper the auto regressive coefficient form of the input audio signal is used as input in the Kalman filter algorithm.By using the Kalman filter the brown noise is removed successfully.the advantage of Kalman filter is zero Gaussian noise.Cascaded Integrator Comb Filter is used as a linear FIR filter to remove the colored noise.

References:

[1] Greg Welch, Garry Bishop, “An Introduction to the Kalman Filter “, Department of Computer Science, University of North Carolina at Chapel Hill.

[2] Maria Isabel Ribeiro, “Kalman and Extended Kalman Filters: Concept, Derivation and Properties ” Institute of Systems and Robotics.

[3] Nari Tanabe , Toshirio Furukawa , Hideaki Matsue , Shigeo Tsujii ,”Noise suppression with High Speech Quality Based on Kalman filter” ,IEEE,2006 International Symposium on Intelligent Signal Processing and Communication Systems(ISPAC 2006), Yonago Convention Center, Tottori, Japan

[4] Nari Tanabe , Toshirio Furukawa , Shigeo Tsujii, “A Kalman Filter based Fast Noise Suppression Algorithm” ,IEEE,Tokyo University of Science , Sua ,Japan.

[5] Anil Singh, Poonam Singhal and Rajeev Ratan “Multistage implementation of multirate CIC filters” ECE,DCRUST , Campus Murthal , Sonepat

Cite this article as: Maitry Chakraborty, Depanwita Sarkar. “Colored Noise Reduction By Kalman Filter.” *International Conference on Information Engineering, Management and Security (2015)*: 43-48. Print.