Cancellation of ISI and High Spectral Efficiency Using Adaptive OFDMA

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Abstract: The demand for high data rate services has been increasing very rapidly and there is no slowdown in sight. Almost every existing physical medium capable of supporting broadband data transmission to our homes, offices and schools has been or will be used in the future. This includes both wired (Digital Subscriber Lines, Cable Modems, Power Lines) and wireless media. Often, these services require very reliable data transmission over very harsh environments. Most of these transmission systems experience many degradations, such as large attenuation, noise, multipath, interference (ISI), low spectral efficiency time variation, non-linearity’s, and must meet many constraints, such as finite transmit power and most importantly finite cost. In such channels, extreme fading of the signal amplitude occurs and Inter Symbol Interference (ISI) due to the frequency selectivity of the channel appears at the receiver side. This leads to a high probability of errors and the system’s overall performance becomes very poor. Adaptive orthogonal frequency division multiple access (OFDMA) has recently been recognized as a promising technique for providing high spectral efficiency by updating SCA subcarrier allocation using slow adaptive OFDMA system. This paper proposes a slow adaptive OFDMA scheme in which the subcarrier allocation is updated on a much slower timescale than that of the fluctuation of instantaneous channel conditions. However, such “fast” adaptation requires high computational complexity and excessive signaling overhead. This hinders the deployment of adaptive OFDMA systems worldwide. We formulate safe tractable constraints for the problem based on recent advances in chance constrained programming. We apply the chance constrained programming methodology to wireless system designs. We then develop a polynomial-time algorithm for computing an optimal solution to the reformulated problem. Our results show that the proposed slow adaptation scheme drastically reduces ISI and improves spectral efficiency when compared with the conventional fast adaptive OFDMA. Our work can be viewed as an initial attempt to apply the chance constrained programming methodology to wireless system designs.

Index terms: Adaptive orthogonal frequency division multiple access (OFDMA), chance constrained programming, dynamic resource allocation, stochastic programming.

I. Introduction

In the existing literature, adaptive OFDMA exploits time, frequency, and multiuser diversity by quickly adapting subcarrier allocation (SCA) to the instantaneous channel state information (CSI) of all users. Such “fast” adaptation suffers from high computational complexity, since an optimization problem required for adaptation has to be solved by the base station (BS) every time the channel changes. Considering the fact that wireless channel fading can vary quickly (e.g., at the order of milliseconds in wireless cellular system), the implementation of fast adaptive OFDMA becomes infeasible for practical systems, even when the number of users is small. Recent work on reducing complexity of fast adaptive OFDMA includes [5], [6], etc. Moreover, fast adaptive OFDMA requires frequent signaling between the BS and the mobile users in order to inform the users of their latest allocation decisions. The overhead thus incurred is likely to negate the performance gain obtained by the fast adaptation schemes. To date, high computational cost and high control signaling overhead are the major hurdles that pre-vent adaptive OFDMA from being deployed in practical systems.

We consider a slow adaptive OFDMA scheme, which is motivated by [7], to address the aforementioned problem. In contrast to the common belief that radio resource allocation should be readapted once the instantaneous channel conditions change, the proposed scheme updates the SCA on a much slower
timescale than that of channel fluctuation.

In this paper, we propose a slow adaptive OFDMA scheme that aims at maximizing the long-term system throughput while satisfying with high probability the short-term data rate requirements. The key contributions of this paper are as follows:

This paper considers a single-cell multiuser OFDM system with $k$ users and $N$ subcarriers. We assume that the instantaneous channel coefficients of user $k$ and subcarriers $n$ are described by complex Gaussian random variables $h_{k,n}^{(t)} \sim \mathcal{C}(0, \sigma_k^2)$, independent in both $n$ and $k$. The parameter $\sigma_k$ can be used to model the long-term average channel gain as $\sigma_k = (d_k / d_0)^{-\gamma} \cdot s_k$, where $d_k$ is the distance between the BS and subscribe $k$, is the reference distance, $\gamma$ is the amplitude path-loss exponent and $s_k$ characterizes the shadowing effect. Hence, the channel gain $g_{k,n}^{(t)}$ is an exponential random variable with probability density function (PDF) given by

$$f_{g_{k,n}}(\xi) = \frac{1}{\sigma_k} \exp \left( \frac{-\xi}{\sigma_k} \right).$$

The transmission rate of user $k$ on subcarrier $n$ at time $t$ is given by

$$r_{k,n}^{(t)} = W \log_2 \left( 1 + \frac{p_t g_{k,n}^{(t)}}{\Gamma N_0} \right)$$

where $p_t$ is the transmission power of a subcarrier, $g_{k,n}^{(t)}$ is the channel gain at time $t$, $W$ is the bandwidth of a subcarrier, $N_0$ is the power spectral density of Gaussian noise, and $\Gamma$ is the capacity gap that is related to the target bit error rate (BER) and coding-modulation schemes.

In traditional fast adaptive OFDMA systems, SCA decisions are made based on instantaneous channel conditions in order to maximize the system throughput. As depicted in Fig. 1(a), SCA is performed at the beginning of each time slot, where the duration of the slot is no larger than the coherence time of the channel. Denoting by $x_{k,n}^{(t)}$ the fraction of airtime assigned to user $k$ on subcarrier $n$, fast adaptive OFDMA solves at each time slot $t$ the following linear programming problem.

![Adaptation timescales of fast and slow adaptive OFDMA system](image-url)

**Fig. 1.** Adaptation timescales of fast and slow adaptive OFDMA system.

(a) Fast adaptive OFDMA. (b) Slow adaptive OFDMA.
where the objective function in (2) represents the total system throughput at time \( t \), and (3) represents the data rate constraint of user \( k \) at time \( t \) with \( \gamma_k \) denoting the minimum required data rate. We assume that \( \gamma_k \) is known by the BS and can be different for each user \( k \). Since \( \mathcal{g}_{k,n}^{(t)} \) (and hence \( \gamma_k^{(t)} \)) varies on the order of coherence time, one has to solve the Problem \( \mathcal{P}_{fast} \) at the beginning of every time slot \( t \) to obtain SCA decisions. Thus, the above fast adaptive OFDMA scheme is extremely costly in practice.

In contrast to fast adaptation schemes, we propose a slow adaptation scheme in which SCA is updated only every adaptation window of length \( T \). More precisely, SCA decision is made at the beginning of each adaptation window as depicted in Fig. 1(b), and the allocation remains unchanged till the next window. We consider the duration \( T \) of a window to be large compared with the fast fading fluctuation so that the channel fading process over the window is ergodic; but small compared with the large-scale channel variation so that path-loss and shadowing are considered to be fixed in each window. Unlike fast adaptive systems that require the exact CSI to perform SCA, slow adaptive OFDMA systems rely only on the distributional information of channel fading and make an SCA decision for each window.

Let \( x_k \in \{0,1\} \) denote the SCA for a given adaptation window. Then, the time-average throughput of user \( k \) during the window becomes

\[
\overline{\eta}_k = \sum_{n=1}^{N} x_{k,n} \overline{r}_{k,n}
\]

Where

\[
\overline{r}_{k,n} = \frac{1}{T} \int_{t_0}^{t_0+T} \mathcal{g}_{k,n}^{(t)} \, dt
\]

is the time-average data rate of user \( k \) on subcarrier \( n \) during the adaptation window. The time-average system throughput is given by

\[
\overline{b} = \sum_{k=1}^{K} \sum_{n=1}^{N} x_{k,n} \overline{r}_{k,n}.
\]

Now, suppose that each user has a short-term data rate requirement \( \eta_k \) defined on each time slot. If \( \sum_{n=1}^{N} x_{k,n} < \eta_k \), then we say that a rate outage occurs for user \( k \) at time slot \( t \), and the probability of rate outage for user \( k \) during the window \([t_0, t_0 + T]\) is defined as
Where $t_0$ is the beginning time of the window.

Inelastic applications, such as voice and multimedia, that are concerned with short-term QoS can often tolerate an occasional dip in the instantaneous data rate. In fact, most applications can run smoothly as long as the short-term data rate requirement is satisfied with sufficiently high probability. With the above considerations, we formulate the slow adaptive OFDMA problem as follows:

$$P_{k, n}^{\text{out}} \doteq \Pr \left\{ \sum_{n=1}^{N} x_{k, n} r_{k, n}^{(t)} < q_k \right\}, \quad \forall t \in [t_0, t_0 + T]$$

Where $t_0$ is the beginning time of the window.

The expectation in (4) is taken over the random channel process $g = \{ g(t) \}$ for $t \in [t_0, t_0 + T]$, and $\epsilon \in [0, 1]$ in (5) is the maximum outage probability user $k$ can tolerate. In the above formulation, we seek the optimal SCA that maximizes the expected system throughput while satisfying each user’s short-term QoS requirement, i.e., the instantaneous data rate of user $k$ is higher than $q_k$ with the probability at least $1 - \epsilon_k$. The above formulation is a chance constrained (5) has been imposed.

II Safe Tractable Constraints

Despite its utility and relevance to real applications, chance constraint (5) imposed in $p_{\text{slow}}$ makes the optimization highly intractable. The main reason is that the convexity of the feasible set defined by (5) is difficult to verify. Indeed, given a generic chance constraint $\Pr\{F(x, r) > 0\} \leq \epsilon$, where $r$ is a random vector, $x$, is a vector of decision variable $F$ is a real valued function, its feasible set is often nonconvex except for very few special cases. Moreover, even with the function in (5) $F(x, r) = q_k - \sum_{n=1}^{N} x_{k, n} r_{k, n}^{(t)}$ is bilinear in $x$ and $r$ whose distribution is known, it is still unclear how to compute the probability in (5) efficiently.

To circumvent the above hurdles, we propose the following formulation $p_{\text{slow}}$ by replacing the chance constraints (5) with system of constraints $H$ such that (i) $x$ is feasible for (5) whenever it is feasible for $H$, $H$ are convex and efficiently unfeasible. The new formulation is given as follows:

$$P_{\text{slow}} : \max_{x, \epsilon} \sum_{k=1}^{K} \sum_{n=1}^{N} x_{k, n} E\left\{ r_{k, n}^{(t)} \right\}$$

s.t. $\inf_{\epsilon > 0} \left\{ q_k + \sum_{n=1}^{N} \lambda_k (-\epsilon^{-1} x_{k, n}) \right\} \leq 0, \quad \forall k$

$$\sum_{k=1}^{K} x_{k, n} \leq 1, \quad \forall n$$

$$x_{k, n} \geq 0, \quad \forall k, n$$
Where $\Lambda_k(\cdot)$ is the cumulant generating function of $x_{k,n}^{(t)}$, and

$$
\Lambda_k(-q^{-1}x_{k,n}) = \log \left[ \int_0^\infty \left( 1 + \frac{p_n \xi}{\Gamma N_0} \right)^\frac{-W_k \xi}{q \ln 2} \cdot \frac{1}{\sigma_k} \exp \left( -\frac{\xi}{\sigma_k} \right) d\xi \right].
$$

In the following, we first prove that any solution $x$ that is feasible for the STC (7) in $p_{\text{slow}}$ is also feasible for the chance constraints. Then we prove that $p_{\text{slow}}$ is convex.

## III Algorithm

### 1 Structure of the Proposed Algorithm

**Require:** The feasible solution set of Problem $P_{\text{slow}}$ is a compact set $\mathcal{X}$ defined by (7)–(9).

1. Construct a polytope $X_0 \supset \mathcal{X}$ by (8) and (9). Set $i \leftarrow 0$.

2. Choose a query point (Section IV-A-1) at the $i$th iteration as $x^i$ by computing the analytic center of $X_i$. Initially, set $x^0 = \bar{c}^T K \in X_0$ where $\bar{c}$ is an $N$-vector of ones.

3. Query the separation oracle (Section IV-A-2) with $x^i$.

4. If $x^i \in \mathcal{X}$ then generate a hyperplane (optimality cut) through $x^i$ to remove the part of $X_i$ that has lower objective values.

5. Else

6. Generate a hyperplane (feasibility cut) through $x^i$ to remove the part of $X_i$ that contains infeasible solutions.

7. end if

8. Set $i \leftarrow i + 1$. and update $X_{i+1}$ by the separation hyperplane.

9. if termination criterion (Section IV-B) is satisfied then

10. stop.

11. else

12. return to step 2.

13. end if

### IV Simulation Results

In this section, we demonstrate the performance of our proposed slow adaptive OFDMA scheme through numerical simulations. We simulate an OFDMA system with four users and 64 subcarriers. Each user $k$ has a requirement on its short-term
V. Conclusion

This paper proposed a slow adaptive OFDMA scheme that can achieve a throughput close to that of fast adaptive OFDMA schemes, while significantly reducing the ISI, computational complexity and control signaling overhead and increases the spectral efficiency. Our scheme can satisfy user data rate requirement with high probability. This is achieved by formulating our problem as a stochastic optimization problem. Based on this formulation, we design a polynomial-time algorithm for subcarrier allocation in slow adaptive OFDMA.

In the future, it would be interesting to investigate the chance constrained subcarrier allocation problem when frequency correlation exists, or when the channel distribution information is not perfectly known at the BS. Moreover, it is worthy to study the tightness of the Bernstein approximation. Another interesting direction is to consider discrete data rate and exclusive subcarrier allocation. In fact, the proposed algorithm based on cutting plane methods can be extended to incorporate integer constraints on the variables.
Finally, our work is an initial attempt to apply the chance constrained programming methodology to wireless system designs. As probabilistic constraints arise quite naturally in many wireless communication systems due to the randomness in channel conditions, user locations, etc., we expect that chance constrained programming will find further applications in the design of high performance wireless systems.

References