Simplified Frequency Offset Estimation in MIMO OFDM Systems

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Abstract—This paper addresses a simplified frequency offset estimator for multiple-input–multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems over frequency-selective fading channels. By exploiting the good correlation property of the training sequences, which are constructed from the Chu sequence, carrier frequency offset (CFO) estimation is obtained through factor decomposition for the derivative of the cost function with great complexity reduction. The mean square error (MSE) of the CFO estimation is derived to optimize the key parameter of the simplified estimator as well as to evaluate the estimator performance. Simulation results confirm the good performance of the training-assisted CFO estimator.

Index Terms—Frequency offset estimation, frequency-selective fading channels, low complexity, multiple-input– multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM).

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I. Introduction

Orthogonal frequency division multiplexing (OFDM) transmission is receiving increasing attention in recent years due to its robustness to frequency-selective fading and its sub carrier-wise adaptability. On the other hand, multiple-input multiple-output (MIMO) systems attract considerable interest due to the higher capacity and spectral efficiency that they can provide in comparison with single-input single-output (SISO) systems. Accordingly, MIMO-OFDM has emerged as a strong candidate for beyond third generation (B3G) mobile wide-band communications [1].It is well known that SISO-OFDM is highly sensitive to carrier frequency offset (CFO), and accurate estimation and compensation of CFO is very important [2]. A number of approaches have dealt with CFO estimation in a SISO-OFDM setup [3], [4], [2], [5], [6], [7]. According to whether the CFO estimators use training sequences or not, they can be classified as blind ones [3] [4] and training-based ones [2],[5], [6], [7]. Similar to SISO-OFDM, MIMO-OFDM is also very sensitive to CFO. Moreover, for MIMO-OFDM, there exists multi-antenna interference (MAI) between the received signals from different transmit antennas. The MAI makes CFO estimation more difficult, and a careful training sequence design is required for training-based CFO estimation. However, unlike SISO-OFDM, only a few works on CFO estimation for MIMO-OFDM have appeared in the literature. In [8], a blind kurtosis-based CFO estimator for MIMO-OFDM was developed. For training-based CFO estimators, the overviews concerning the necessary changes to the training sequences and the corresponding CFO estimators when extending SISO-OFDM to MIMO-OFDM were provided in [9], [10]. However, with the provided training sequences in [9], satisfactory CFO estimation performance cannot be achieved. With the training sequences in [10], the training period grows linearly with the number of transmit antennas, which results in an increased overhead. In [11], a white sequence based maximum likelihood (ML) CFO estimator was addressed for MIMO, while a hopping pilot based CFO estimator was proposed for MIMO-OFDM in [12]. Numerical calculations of the CFO estimators in [11] [12] require a large point discrete Fourier transform (DFT) operation and a time consuming line search over a large set of frequency grids, which make the estimation computationally prohibitive. To reduce complexity, computationally efficient CFO estimation was introduced in [13] by exploiting proper approximations. However, the CFO estimator in [13] is only applied to flat-fading MIMO channels.

When training sequence design for CFO estimation is concerned, it has received relatively little attention. It was investigated for single antenna systems in [14], where a white sequence was found to minimize the worst-case asymptotic Cramer-Rao bound (CRB). Recently, an improved training sequence and structure design was developed in [15] by exploiting the CRB and received training signal statistics. In [16], training sequences were designed for CFO estimation in MIMO systems using a channel-independent CRB. In [17], the effect of CFO was incorporated into the mean square error (MSE) optimal training sequence designs for MIMO-OFDM channel estimation in [18]. Note that optimal training sequence design for MIMO-OFDM CFO estimation in frequency selective fading channels is still an open problem.

In this paper, by further investigating the above search-free approaches, a simplified CFO estimator is developed for multiple input– multiple - output (MIMO) OFDM systems over frequency selective fading channels. With the aid of the training sequences generated from the Chu sequence [9], we propose to estimate the CFO via a simple polynomial factor. Thus, the complicated polynomial rooting operation is avoided. Correspondingly, the CFO estimator can be implemented via simple additions and multiplications. To optimize the key parameter of the simplified CFO estimator as well as to evaluate the estimator performance, the mean square error (MSE) of the CFO estimation is derived

Notations: Upper (lower) bold-face letters are used for matrices (column vectors). Superscripts , *T* and *H* denote conjugate, transpose and Hermitian transpose, respectively. $(\cdot)P$ denotes the residue of the number within the brackets, denote the floor, Euclidean norm-square, expectation and Kronecker product operators, respectively. sign(·) denotes the signum function and sign(0) = 1 is assumed. $[\boldsymbol{x}]m$ denotes the *m*-th entry of a column vector $\boldsymbol{x}$. $\boldsymbol{x}(m)$ denotes the *m*cyclic-down-shift version of $\boldsymbol{x}$. diag$\{\boldsymbol{x}\}$ denotes a diagonal matrix with the elements of $\boldsymbol{x}$ on its diagonal. $[\boldsymbol{X}]m,n$ denote the (m, n)-th entry of a matrix $\boldsymbol{X}$. $\boldsymbol{F}N$ and $\boldsymbol{I}N$ denote the $N \times N$ unitary DFT matrix and the $N \times N$ identity matrix, respectively. $\boldsymbol{E}k\ N$ denotes the *k*-th column vector of IN. $1Q$ ($0Q$) and $0P\times Q$ denote the $Q\times 1$ all-one (all-zero) vector and $P \times Q$ all-zero matrix, respectively. JQ denotes the $Q \times Q$ exchange matrix with ones on its anti-diagonal and zeros elsewhere. Unless otherwise stated, $0 \le \mu \le Nt - 1$ and $0 \le \nu \le Nr - 1$ are assumed

II. Signal Model

Consider a MIMO OFDM system with *Nt* transmit antennas, *Nr* receive antennas, and *N* sub carriers. The training sequences for CFO estimation are the same as in [6] and [7]. Let $\boldsymbol{s}$ denote a length-*P* Chu sequence [9]. Then, the $P \times 1$ pilot sequence vector at the μth transmit antenna is generated from $\boldsymbol{s}$ as follows:

$\tilde{s}_\mu = \sqrt{Q/N_t}\boldsymbol{F}_P s^{(\mu M}$, where $M = \lfloor P/Nt \rfloor$ Define

$$\boldsymbol{\Theta}_q = [\boldsymbol{e}_N^q, \boldsymbol{e}_N^{q+Q}, \ldots, \boldsymbol{e}_N^{q+(P-1)Q}]$$

Then, the $N \times 1$ training sequence vector at the μth transmit antenna is constructed as follows:

Let $\boldsymbol{y}_\nu$ denote the $N \times 1$ received vector at the νth receive antenna after cyclic prefix (CP) removal. Let $\boldsymbol{h}(\nu,\mu)$ denote the $L \times 1$ channel impulse response

$\tilde{t}_\mu = \Theta_{i_\mu}\tilde{s}_\mu$, where $0 \le i_\mu \le Q-1$, and $i_\mu = i_{\mu'}$ iff $\mu = \mu'$. For convenience, we refer to $\{\tilde{t}_\mu\}_{\mu=0}^{N_t-1}$ as the Chu-sequence-based training sequences (CBTSs).

vector, with *L* being the maximum channel length. Assume that *L* is shorter than the length of CP *Ng*. Let ˜ε denote the frequency offset normalized by the subcarrier frequency spacing. Define

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$$y = \left[y_0^T, y_1^T, \ldots, y_\nu^T, \ldots, y_{N_r-1}^T\right]^T$$
$$h_\nu = \left[\left(h^{(\nu,0)}\right)^T, \left(h^{(\nu,1)}\right)^T, \ldots, \left(h^{(\nu,\mu)}\right)^T, \ldots \left(h^{(\nu,N_t-1)}\right)^T\right]^T$$
$$h = \left[h_0^T, h_1^T, \ldots, h_\nu^T, \ldots, h_{N_r-1}^T\right]^T$$
$$D_N(\tilde{\varepsilon}) = \text{diag}\left\{\left[1, e^{j2\pi\tilde{\varepsilon}/N}, \ldots, e^{j2\pi\tilde{\varepsilon}(\tilde{N}-1)/N}\right]^T\right\}.$$

Then, the cascaded received vector **y** over the *Nr* receive antennas can be written as [6], [7]

$$y = \sqrt{N} e^{j2\pi\tilde{\varepsilon}N_g/N} \left\{I_{N_r} \otimes [D_N(\tilde{\varepsilon})S]\right\} h + w \qquad (1)$$

Where

$$S = F^H \text{diag}\left\{\left[\tilde{s}_0^T, \tilde{s}_1^T, \ldots, \tilde{s}_\mu^T, \ldots, \tilde{s}_{N_t-1}^T\right]^T\right\} \check{F}$$
$$F = \left[\Theta_{i_0}, \Theta_{i_1}, \ldots, \Theta_{i_\mu}, \ldots, \Theta_{i_{N_t-1}}\right]^T F_N$$
$$\check{F} = \left[e_{N_t}^0 \otimes \Theta_{i_0}^T, e_{N_t}^1 \otimes \Theta_{i_1}^T, \ldots, e_{N_t}^\mu \otimes \Theta_{i_\mu}^T, \ldots, e_{N_t}^{N_t-1} \otimes \Theta_{i_{N_t-1}}^T\right]\left\{I_{N_t} \otimes \left[F_N \left[I_L, 0_{L\times(N-L)}\right]^T\right]\right\}$$

and ***w*** is an *NrN* × 1 vector of uncorrelated complex Gaussian noise samples with a mean of zero and an equal variance of σ2*w*

III. CFO Estimator for MIMO-OFDM Systems

By exploiting the periodicity property of CBTS, **y** can be stacked into the *Q* × *NrP* matrix

Y = [***Y*** 0,***Y*** 1, . . . , ***Y*** ν, . . . ,***Y*** *Nr*– 1] with its element given by

$$[Y_\nu]_{q,p} = [((e_{N_r}^\nu)^T \otimes I_N) y]_{qP+p}$$

define

$$b_\mu = \left[1, e^{j2\pi(\tilde{\varepsilon}+i_\mu)/Q}, \ldots, e^{j2\pi(\tilde{\varepsilon}+i_\mu)q/Q}, \ldots, e^{j2\pi(\tilde{\varepsilon}+i_\mu)(Q-1)/Q}\right]^T$$
$$B(\tilde{\varepsilon}) = [b_0, b_1, \ldots, b_\mu, \ldots, b_{N_t-1}].$$

Then, ***Y*** can be expressed in the following equivalent form [6], [7]:

$$Y = B(\tilde{\varepsilon})X + W \qquad (2)$$

where

$$X = [X_0, X_1, \ldots, X_\nu, \ldots, X_{N_r-1}]$$
$$X_\nu = \left[x^{(\nu,0)}, x^{(\nu,1)}, \ldots, x^{(\nu,\mu)}, \ldots, x^{(\nu,N_t-1)}\right]^T$$
$$x^{(\nu,\mu)} = \sqrt{P} e^{j2\pi\tilde{\varepsilon}N_g/N} D_P(\tilde{\varepsilon}+i_\mu) \times F_P^H \text{diag}\{\tilde{s}_\mu\} \Theta_{i_\mu}^T F_N \left[I_L, 0_{L\times(N-L)}\right]^T h^{(\nu,\mu)}$$

and ***W*** is the *Q* × *NrP* matrix generated from ***w*** in the same way as ***Y*** According to the multivariate statistical theory, the log-likelihood function of ***Y*** conditioned on ***B***(ε) and ***X***, with ε denoting a candidate CFO, can be obtained as follows:

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$$\ln p\left(\boldsymbol{Y} \mid \boldsymbol{B}(\varepsilon), \boldsymbol{X}\right)=-\sigma_w^{-2} \operatorname{Tr}\left\{[\boldsymbol{Y}-\boldsymbol{B}(\varepsilon) \boldsymbol{X}][\boldsymbol{Y}-\boldsymbol{B}(\varepsilon) \boldsymbol{X}]^H\right\}. \quad (3)$$

Exploit the condition $i\mu = i\mu\_$ iff $\mu = \mu\_$. Then, after some straightforward manipulations, we can obtain the reformulated log-likelihood function conditioned on ε as follows:

$$\ln p(\boldsymbol{Y} \mid \varepsilon)=\operatorname{Tr}\left[\boldsymbol{B}^H(\varepsilon) \hat{\boldsymbol{R}}_{\boldsymbol{Y}\boldsymbol{Y}} \boldsymbol{B}(\varepsilon)\right] \quad (4)$$

where $R_{YY} = \boldsymbol{Y}\boldsymbol{Y}^H$. Direct grid searching from (4)

Yields the ML estimate; however, this approach is computationally quite expensive. To efficiently compute the CFO, we will subsequently propose a simplified CFO estimator for MIMO OFDM systems. Define $z = e^{j2\pi\varepsilon/Q}$, $z\mu = e^{j2\pi i\mu/Q}$, and $\boldsymbol{b}(z) = [1, z, \ldots, zq, \ldots, zQ-1]T$. Then, by exploiting the Hermitian property of, the log-likelihood function in (4) $^R Y\ Y$ can be transformed into the following equivalent form:

Where

$$f(z)=\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}(z_\mu)\right] \odot \boldsymbol{b}(z)\right\} + \boldsymbol{c}^H\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}\left(z_\mu^{-1}\right)\right] \odot \boldsymbol{b}(z^{-1})\right\} \quad (5)$$

Where $\boldsymbol{c}$ is a $Q \times 1$ vector with its qth element given by $[\boldsymbol{c}]q = \sum j-i = q[\hat{\ } \boldsymbol{R}_{YY}]i,j$. It can be seen from its definition that the qth element of $\boldsymbol{c}$ corresponds to the summation of the qth upper diagonal elements of ^{R} $\boldsymbol{Y}\boldsymbol{Y}$ Taking the first-order derivative of $f(z)$ with respect to z yields

$$f'(z)=z^{-1}\left\{\boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}(z_\mu)\right] \odot \boldsymbol{b}(z) \odot \boldsymbol{q}\right\} - \boldsymbol{c}^H\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}\left(z_\mu^{-1}\right)\right] \odot \boldsymbol{b}(z^{-1}) \odot \boldsymbol{q}\right\}\right\} \quad (6)$$

where $\boldsymbol{q} = [0, 1, \ldots, q, \ldots, Q-1]T$. By letting the derivative of the log-likelihood function $f\_(z)$ be zero, the solutions for all local minima or maxima can be obtained. Put these solutions back into the original log-likelihood function $f(z)$, and select the maximum by comparing all the solutions obtained in the previous stage. The improved blind CFO estimator that exploits the above mathematical rule has been addressed for single-antenna OFDM systems in [3]. Although the search-free approach has a relatively lower complexity, it still requires a complicated polynomial rooting operation, which is hard to implement in practical OFDM systems. With the aid of the CBTS training sequences, we will show in the following that the polynomial rooting operation can be avoided for training-aided CFO estimation in MIMO OFDM systems. Assume that $P \geq L$, the channel taps remain constant during the training period, and the channel energy is mainly concentrated in the first M taps, with $M < L$. Then

$$\boldsymbol{c}^H\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}\left(z_\mu^{-1}\right)\right] \odot \boldsymbol{b}(z^{-1}) \odot \boldsymbol{q}\right\} = z^{-Q}\kappa(\iota) \cdot \boldsymbol{c}^T\left\{\left[\sum_{\mu=0}^{N_t-1} \boldsymbol{b}(z_\mu)\right] \odot \boldsymbol{b}(z) \odot \boldsymbol{q}\right\} \quad (7)$$

with $1 \leq \iota \leq Q-1$, and the parameter ι denotes the index of the upper diagonal of . From (7), it ^{R} $\boldsymbol{Y}\ \boldsymbol{Y}$ immediately follows that $f'(z)$ can be decomposed as follows:

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$$f'(z)=z^{-(Q+1)}\left[z^{Q}-\kappa(\iota)\right]\cdot \mathbf{c}^{T}\left\{\left[\sum_{\mu=0}^{N_t-1} \mathbf{b}(z_\mu)\right]\odot \mathbf{b}(z)\odot \mathbf{q}\right\}. \quad (8)$$

It follows from (8) and (9) that $z = \tilde{z}$ is one of the roots of both $f'(\tilde{z}) = 0$ and $zQ - \kappa(\iota) = 0$. Unlike $f\_(\tilde{z}) = 0$, the roots of $zQ - \kappa(\iota) = 0$ can be calculated without the polynomial rooting operation. Therefore, by solving the simple polynomial equation $zQ - \kappa(\iota) = 0$, the CFO estimate can be efficiently obtained as follows:

$$\hat{\varepsilon} = \underset{\varepsilon\in\{\varepsilon_q\}_{q=0}^{Q-1}}{\arg\max}\left\{f(z)|z = e^{j2\pi\varepsilon/Q}\right\} \quad (10)$$

Where $\varepsilon q = \arg\{\kappa(\iota)\}/(2\pi) + q - Q/2$. It can be calculated that the main computational complexity of the simplified CFO estimator is $4NrNQ + 8Q2$. Compared with the CFO estimator in [6] and [7], whose main computational complexity is $4NrN \log 2\ N + 9Q3 + 64/3(Q - 1)3$, the complexity of the simplified CFO estimator is generally lower. Furthermore, since the polynomial rooting operation is avoided, the simplified CFO estimator can be implemented via simple additions and multiplications, which is more suitable for practical OFDM systems. Note that ι is a key parameter for the proposed CFO estimator. We will show in the following how to determine the optimal ι.

IV. Simulation Results

Numerical results are provided to verify the analytical results as well as to evaluate the performance of the proposed CFO estimator. The considered MIMO OFDM system has a bandwidth of 20 MHz and a carrier frequency of 5 GHz with $N = 1024$ and $Ng = 80$. Each of the channels is with six independent Rayleigh fading taps, whose relative average powers and propagation delays are {0,− 0.9,− 4.9,− 8.0,− 7.8,− 23.9} dB and {0, 4, 16, 24, 46, 74} samples, respectively. The other parameters are given as follows: $P = 64$, $Q = 16$, $Nt = 3$, $Nr = 2$, and $\tilde{\varepsilon} \in (-Q/2, Q/2)$. Figs. 1 and 2 present the MSE of the proposed CFO estimator as a function of ι with $\{i\mu\}Nt-1\ \mu=0 = \{3, 5, 11\}$ and $\{3, 7, 14\}$, respectively. The solid and dotted curves are the results from the analysis and Monte Carlo simulations, respectively. It can be observed that the results from the analysis agree quite well with those from the simulations, except when the actual MSE of the estimate is very large. It can also be observed that MSE$\{\hat{\varepsilon}\}$ achieves its minimum for ι = 6, 8, 10 with $\{i\mu\}Nt-1\ \mu=0 = \{3, 5, 11\}$ and for ι = 7, 9 with $\{i\mu\}Nt-1\ \mu=0 = \{3, 7, 14\}$. These observations imply that we can obtain the optimum value of the parameter ι from the analytical results after $\{i\mu\}Nt-1\ \mu=0$ is determined

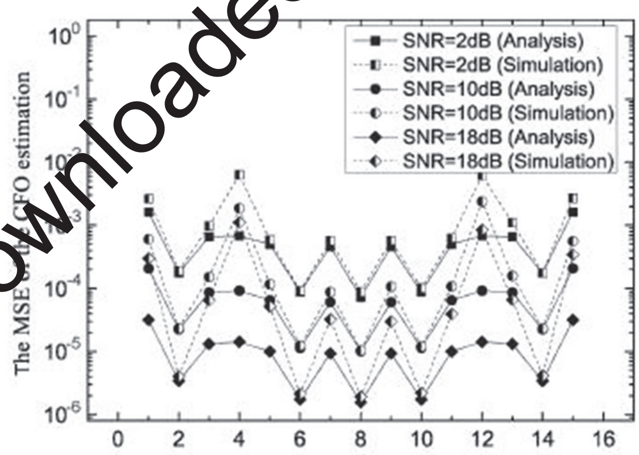

Fig. 1. MSE of the proposed CFO estimator as a function of ι with $_i\ Nt\ 1 = \{3, 5, 11\}$.

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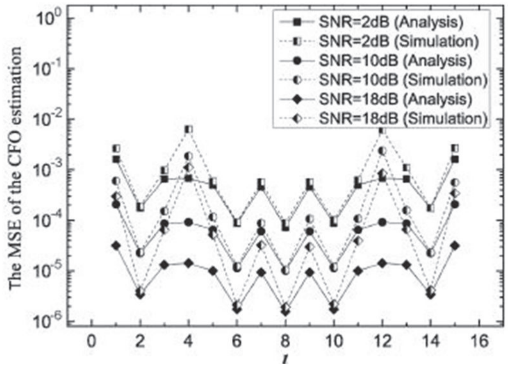

Fig. 2. MSE of the proposed CFO estimator as a function of ι
with $i^{Nt\ 1}$ ={3,7,14}.
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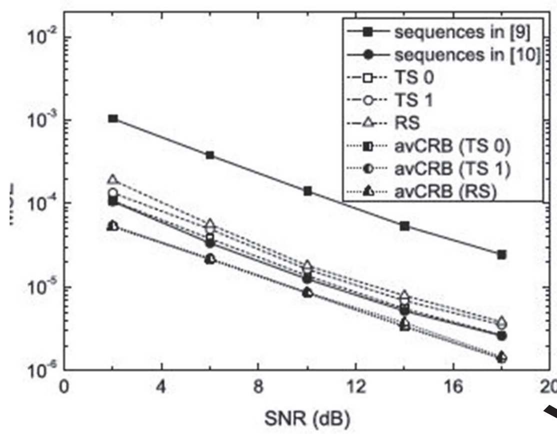

Fig. 1. CFO estimation performance for different training sequences with Nt = 3 and Nr = 2.

We resort to Monte Carlo simulation for its evaluation. It can be observed that the performance of the proposed estimator with CBTS is far better than that in [10] and slightly worse than that in [6] and [7], and its performance also approaches the EMCB, which verifies its high estimation accuracy. It can also be observed that the performance of the proposed CFO estimator with CBTS is far better than that with RS, which should be attributed to the good correlation property of CBTS.

V. Conclusion

In this paper, we have presented a low-complexity CFO estimator for MIMO OFDM systems with the training sequences generated from the Chu sequence. TheMSE of the CFO estimation has been developed to evaluate the estimator performance and to optimize the key parameter. By exploiting the optimized parameter from the estimation MSE, our CFO estimator with CBTS yields good performance

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