

# FEA Analysis for Investigation of Stress Intensity Factor (SIF) for a Plate with Hole and Patches

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**Abstract:** Material and its processing effect the relative structure contains small flaws whose size and distribution. These may vary from nonmetallic inclusions and micro voids to weld defects, grinding cracks, quench cracks, surface laps, etc. The objective of a Fracture Mechanics analysis is to determine if these small flaws will grow into large enough cracks to cause the component to fail catastrophically. By using fracture mechanics theories i.e. LEFM approach, applied to two dimensional objects such as plates and shells etc. The aim of this paper is to estimate numerically the beneficial effect of a plate with hole and bounded single and double side patches are symmetrically bonded on the top and bottom on surface for repairing crack using the finite element method. An observed the change in stress intensity factor along the crack length by using FRANC-2D/L simulation software. The obtained results show the reduction of stress intensity factors. The effects of the adhesive, patch and plate properties on the rate of reduction of the stress intensity factors are highlighted.

**Keywords:** Stress intensity factor, plate, patches, finite element method.

## 1. Introduction

The determination of fracture mechanics parameters such as Stress Intensity Factors (SIF) ( $K_I$ ,  $K_{II}$ ,  $K_{III}$ ) which corresponds to three basic modes of fracture.  $J$ -Integral a path-independent line that measure the strength of the singular stresses and strains near a crack tip. Energy release rate ( $G$ ) which represents the amount of work associated with a Crack Opening Displacement (COD) or Closure. By comparing these parameters with critical values one can estimate the brittle failure state of structures. The crack analysis is found on three different following methods [2]. Which are displacement correlation methods, virtual crack extension methods, and modified crack closure method. The FRANC-2D/L simulation software is based on Finite element method (FEM) and boundary element method (BEM) are the most widely use techniques for evaluating stress intensity factor (SIF). The most important region in modeling the fracture is around the crack. While the domain is meshed we are using crack tip elements with singularity [16, 17], those elements remove the nodal singularity at the crack tip. Displacement correlation method was employed. To determine stress intensity factors (SIF)

The main objective of this paper is the stress intensity factors are calculated from displacement correlation method.

## 1. Displacement Correlation Method

Displacement correlation is a direct approach which the simplest and historically one of the first technique used to estimate SIFs from FEM results [3]. In its simplest form, the finite element displacements for one point in the same mesh are substituted directly into the analytical expressions for near tip displacements, after subtracting the displacements of the crack tip [1].

The point is to be selected as a node on the crack face where the displacements will be greatest, and thus the relative error in the displacements is expected to be smallest. The configuration for this simple approach is shown in fig 1.

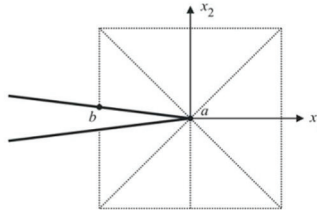


Fig.1. Possible sample point locations for displacement correlation.

The expressions for the SIF's using plane strain assumptions are:

$$K_{I} = \frac{\mu\sqrt{2\pi}(v_b - v_a)}{\sqrt{r}(2 - 2\nu)} \quad K_{II} = \frac{\mu\sqrt{2\pi}(u_b - u_a)}{\sqrt{r}(2 - 2\nu)}$$

$$K_{III} = \frac{\mu\sqrt{\pi}(w_b - w_a)}{\sqrt{2r}}$$

The same expressions can be used for plane stress assumptions of  $\nu$  is replaced with:

$$\nu = \frac{\nu}{1 + \nu}$$

Crack tips produce a  $\frac{1}{\sqrt{r}}$  singularity. The stress fields near a crack tip of an isotropic linear elastic material can be expressed as a product of  $\frac{1}{\sqrt{r}}$  and a function of  $\theta$  with a scaling factor K:

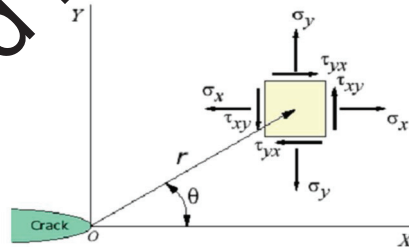


Fig .2. Stress field near crack tip of an isotropic linear elastic material.

$$\lim_{y \rightarrow 0} \sigma_{ij}^{(I)} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{(I)}(\theta)$$

$$\lim_{y \rightarrow 0} \sigma_{ij}^{(II)} = \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{(II)}(\theta)$$

$$\lim_{y \rightarrow 0} \sigma_{ij}^{(III)} = \frac{K_{III}}{\sqrt{2\pi r}} f_{ij}^{(III)}(\theta)$$

### 2. Problem Outline

The objective of this work is to illustrate the process of finite element program to compute the stress intensity factor histories accurately so that they can be used for fatigue crack growth (FCGR) and life predictions.

Mode I (Tension, Opening)

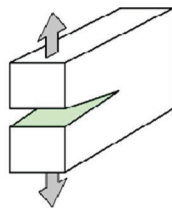


Fig.4. Opening mode of deformation.

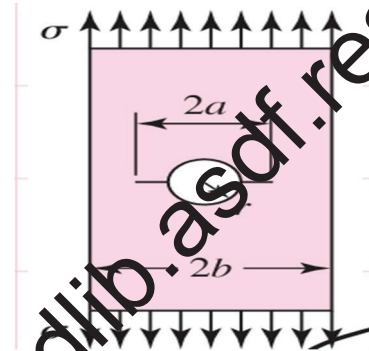


Fig.5. Finite width plate with hole

Fig.5 shows a simple plate with hole under symmetric loading. Let us consider half portion of the Figure (considering the right half). The traction load is applied on the top and bottom edges of the plate. A crack is initiated at the location of high effective stresses, and allows it to propagate in a straight line manner. FRANC 2D/L (Cornell Fracture Group, 2002) [6] is used as the finite element solver.

The input parameters are:

Young's modulus (E) = 10.06E6 N/mm<sup>2</sup>

Poisson's ratio ( $\nu$ ) = 0.25

Thickness of the plate (t) = 0.04mm

Radius of the hole (r) = 2.5mm

Width of the plate (2b) = 4mm

Height of the plate = 8mm

Assumptions are: Material is linearly isotropic,

The problem is modeled as a Plane stress condition.

### 3. Approach

As discussed, the problem is reduced based on symmetry, the plate is modeled by using CASCA [7], and then a converged mesh is made with 8-noded quadrilateral element as shown in fig7. A traction load of 10000N is applied along the top and bottom edges of the plate. This is the condition for the simple plate with hole without addition of any patches and the results are as shown in table1, which will be discussed in results and discussions.

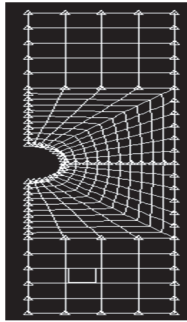


Fig.6. Mesh for plate

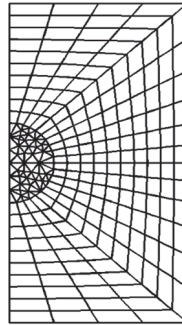


Fig.7. Patch mesh

The mesh for patch after adding patch is as shown in fig 8. The different layers patch and plate after adding adhesive are as shown in fig 9 and 10. The present case is the case for the plate with hole with single patch. In this case on patch is added at the bottom of plate.

For double patch, the patches are added at the top and bottom sides of the plate.

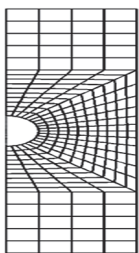


Fig.8. Plate Layer

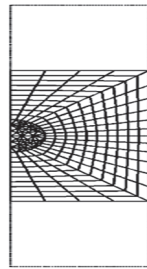


Fig.9. Patch Layer

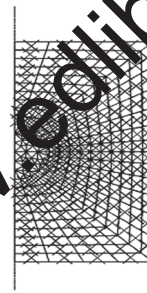


Fig.11. Adhesive elements

**Results and Discussions:**

The following table shows the values of SIF obtained for plate with hole without any patches

Table1. Shows the results for plate with hole with single patch.

sl no	crack length (mm)	$N/mm^2 \cdot mm^{1/2}$
1	0.5	0
2	0.6	14660
3	0.8	19300
4	1	22710
5	1.2	27170
6	1.4	33260

Table2. Shown below shows the results for plate with hole with single patch

sl no	crack length (mm)	$N/mm^2 \cdot mm^{1/2}$
1	0.5	4246

2	0.6	4155
3	0.8	4247
4	1	4256
5	1.2	4372

Table3. SIF values for plate with hole with double patch.

sl no	crack length (mm)	N/mm <sup>2</sup> -mm <sup>1/2</sup>
1	0.5	3094
2	0.7	3030
3	0.9	3096
4	1.1	3141
5	1.4	3468

The following graph shows the variation of SIF along the crack length for the above three cases discussed.

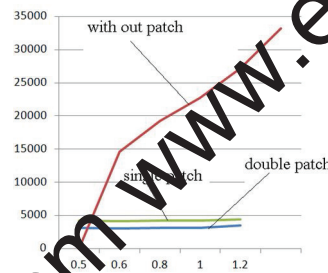


Fig6: Variation of stress intensity factor V/s crack length

The following graph is obtained from experimental observations.

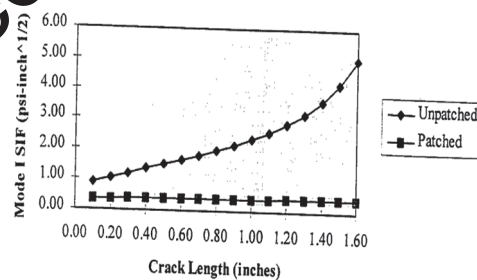


Fig 13: Variations of the stress intensity factors (experimental results)

### Conclusions

In this work The problem is considered under mode-I conditions and  $K_I$  the stress intensity factors are calculated from displacement correlation method .An observed the change in stress intensity factor along the crack length by using FRANC-2D/L simulation software for a plate with hole and bounded single and double side patches are symmetrically bonded on the top and bottom on surface for repairing crack using

the finite element method. The increasing the number of patch layers reduces the stress intensity factors which mean a safe design. The results are plotted by varying crack length are compared with the experimental graphs .The obtained results are found to be satisfactory

### References

1. ASME Meeting Reports on Fracture Mechanics.
2. Paulo F.P. De Matos, Pedro M.G.P. Moreira, Paulo M.S.T. De Castro. "Stress Intensity Factor Determination Using The Finite Element Method." Project GRD1-2000-25069 of the Commission of the European community.
3. Maksimovic, S., Burzic, Z. and Maksimovic, K., fatigue life estimation of notched structural components: Computation and Experimental Investigations , Proc 16 th European Conference of Fracture (ECF 16), Alexandroupolis, Greece, July 3-7, Ed. E.E. Gdoutos, Springer 2006.
4. R. D. Henshell and K G Shaw, "Crack Tip Finite Elements are Unnecessary," International Journal of Numerical Methods in Engineering, Vol 9, pp 495-507, 1975.
5. R.S. Barsoum, "On the use of Isoparametric Finite Elements in Linear Fracture Mechanics," International Journal of Numerical Methods in Engineering. Vol 10, pp 2597, 1976.
6. Daniel Swenson and Mark James FRANC-2D/L: A Crack Propagation Simulator for Plane Layered Structures, Kansas State University, Manhattan, Kansas.
7. Paul Wawrzyneck and Louis Martha (1997) "CASCA: A Simple 2-D Mesh Generator", Cornell University, Ithaca, New York.
8. Shigley's Mechanical Engineering Design, eighth edition, Mc.Graw Hill publication.