Numerical Simulation of Inviscid Flow

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Abstract: Numerical simulation for 1 d and 2 d solvers has been developed by programming to simulate a range of numerical values of the flow properties and has been plotted to visualize various flow characteristics for both 1 d and 2 d cases. Various computational techniques has been employed and cases such as 1 d wave simulation,1 d flow in a pipe (both mach<1.0 and mach>1.0),2 d heat conduction(laplase),2 d flow in a pipe (both mach <1.0 and mach>1.0),2 d flow over a ramp(both mach <1.0 and mach>1.0),2 d flow visualization of shock impinging on a flat plate,2 d flow in a convergent duct(both mach <1.0 and mach>1.0),2 d flow in a divergent duct(both mach <1.0 and mach>1.0),2 d flow over a concave corner(both mach <1.0 and mach>1.0),has been visualized.

Keywords: FVM, CFL, Scheme, Gnuplot, Mayavi, Shock, Ramp, Concave Corner, Prandtl -Meyer.

Introduction

This paper is basically a programming paper that has been developed to bring a "virtual wind tunnel" using the latest Linux programmer called" Ubuntu 10.10 operating system". The greatest advantage of this os is that there is no virus attack. This software is licensed evidently.

Tools used: Gedit/kate editor for coding instead of notepad, terminal as a command prompt for compiling/editing, gnu plot/xmgrace as a 2d plotting tool, mayavi 2 as a3d visualization tool, kile to write a report(technical editor)

Outline of the Thesis: My aim is to develop a 2 d solver and check the refinity of the solver by solving some problems.

Abbreviations: 2d- two dimensional, 3d -three dimensional, fem -finite element method, fvm -finite volume method, fdm- finite difference method, cfl- counrant friedrich lewy, ftfs -forward time forward scheme, ftbs- forward time backward scheme, ftcs- forward time central scheme.

Chapter 1

Numerical Simulation of 1 d Wave Equation here i have used a second order linear partial differential equation for describing the waves. The wave equation is, "(1)"

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

Here I have made use of fdm method and truncation error is said to occur that should be equal to one and the schemes used to test the error are as follows,"(2)","(3)","(4)"

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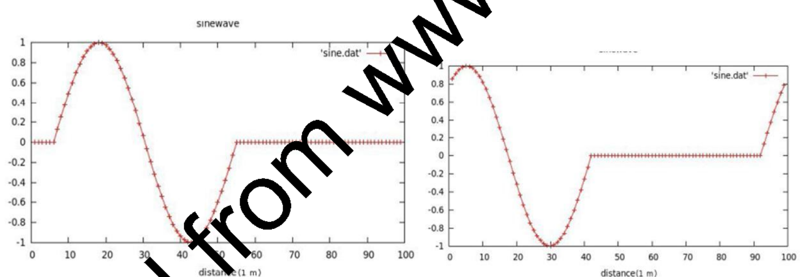
$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x}(\text{Central})$$

$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i}}{\Delta x}(\text{Forward})$$

$$\frac{\partial u}{\partial x} = \frac{u_{i} - u_{i-1}}{\Delta x}(\text{backward})$$

Boundary Conditions

Case (1): For FTCS, u (0) = 0 (at all time) u (n + 1) = 0 (at all time) Case (2): For FTBS, u (0) = 0
Case (3): For FTFS, u (n + 1) = 0
Domain Length is given as _x = l
N−1 where,
l = domain length.



Time marching of sine wave graphical representation

Result

Thus sine wave is generated and a time marching of the sine wave has been achieved and plotted using gnu plot.

Chapter 2

Numerical simulation of 2 d Laplace equation

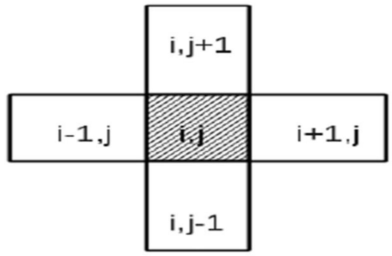
Here the study of heat conduction over a domain is done using Laplace equation. The equation is ,"(5)"

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}$$

We have certain discretization techniques in a symmetrical domain in laplace equation,"(6)"

$$\phi_{ij} = \frac{\phi_{i+ij} + \phi_{i-ij} + \phi_{ij-i} + \phi_{ij-i}}{4}$$

Cell Position Diagram

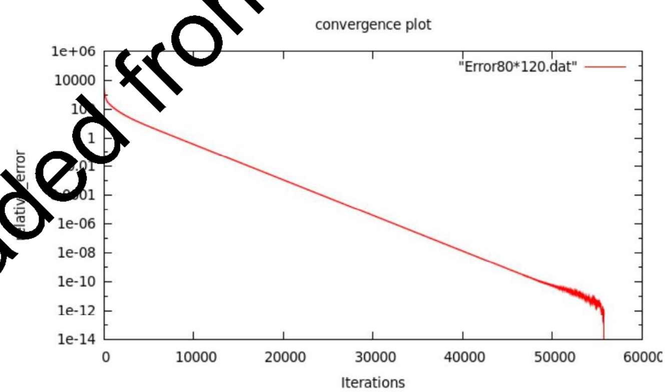


Convergence

The value of inner cells are iterated that the error gets nullified, the error in the iteration is found out using the formula,"(7)"

$$L_2 = \sqrt{\Sigma_{i,j}^{N_x,N_y}(\phi^{new} - \phi)^2}$$

The system is said to converge when the value of l2 falls to 10^-6. It may take several iterations to reach this state. The no: of iterations is directly proportional to the number of grids and solution is checked by plotting.



Chapter 3

Developing a 1 d solver

Here a 1 d solver is developed using 1d eulers equation.

Eulers equation

This are the set of equation governing invicid flow, the equation represent conservation of mass (continuity), momentum, energy,"(8)"

$$\int \frac{\partial Q}{\partial t} dv = - \oint \vec{F}.\hat{n}.ds \text{ (Euler Equation)}$$

Grid Generatio

Here 1d Grid Is Used But For Visualization Purposes We Extend Y Direction

Schemes

Lax-fried rich scheme is used for calculating new primitive variables,"(9)"

$$Q_i^n + 1 = Q_i^n - \frac{\Delta t}{\Delta x}(F_{i+1/2} - F_{i-1/2})$$

Convergence

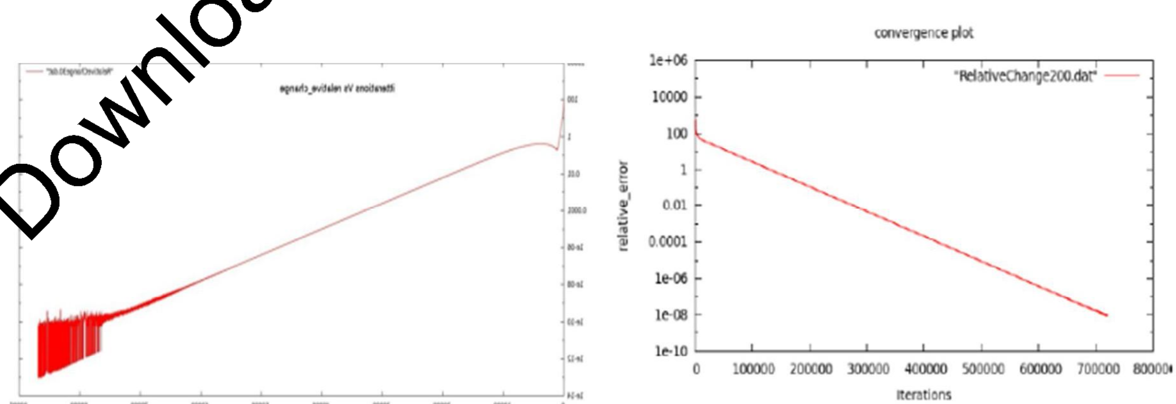
This system is said to converge when there is change in error between the current and previous iteration and converges at the order of 10^-16,"(10)"

$$L_2 = \sqrt{\Sigma_{i,j}^{N_x,N_y} (\phi^{new} - \phi)^2}$$

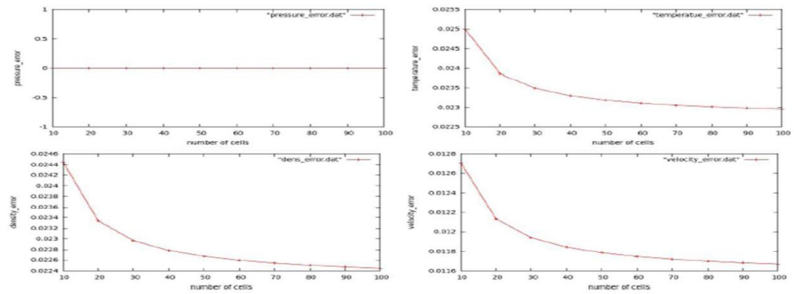
Flux is generated by,"(11)","(12)"

$$F_{i+i/2} = \frac{1}{2}(F_i + F_i + 1) - \frac{1}{2\lambda}(Q_{i+1} - Q_i)$$

$$F_{i-i/2} = \frac{1}{2}(F_{i-1} + F_i) - \frac{1}{2\lambda}(Q_i - Q_{i-1})$$

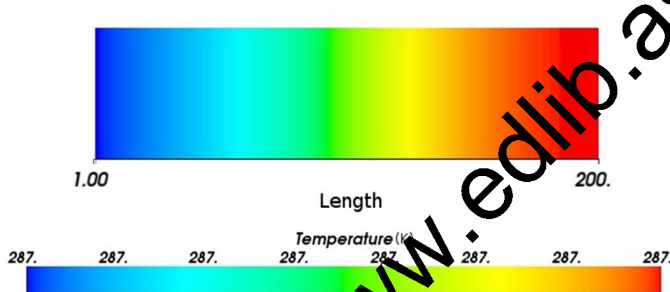


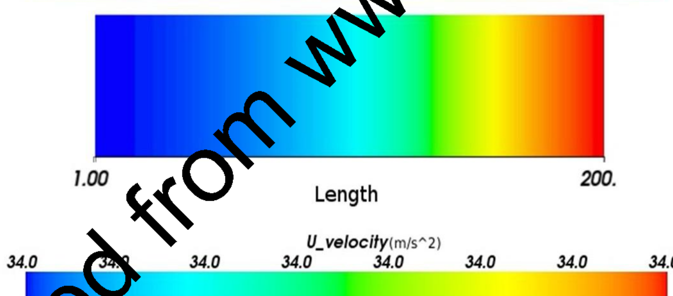
Convergence plot for subsonic and supersonic flow



a)Pressure error b)Temperature error c)Density error d)Velocity error (clock-wise from top)

Result: The solver is tested for mach number 0.1 and 1.5 and is found to converge.





These are the velocity, and temperature solver results.

Chapter 4:

Developing a 2d solver using 2d eulers equation

Eulers Equation

These are set of equations governing inviscid flow, this equation represent conservation of mass (continuity), momentum,energy,"(13)"

$$\int \frac{\partial Q}{\partial t} dv = - \oint \vec{F}.\hat{n}.ds$$

Where,

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E_t \end{bmatrix} E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (\rho E_t + p)u \end{bmatrix} F = \begin{bmatrix} \rho u \\ \rho uv \\ \rho v^2 + p \\ (\rho E_t + p)v \end{bmatrix}$$

Grid Generation

In 2d grid are measured in x and y direction. The domain is equally splitted in both x and y direction. Cell center discritionization is done. It is a solid adiabatic wall.

Boundary Conditions

The properties of wall is made as an average of domain cell and ghost cel,"(14)","(15)".

$$u_{Ghost} = u - 2(\vec{V}.\hat{n}).n_x$$

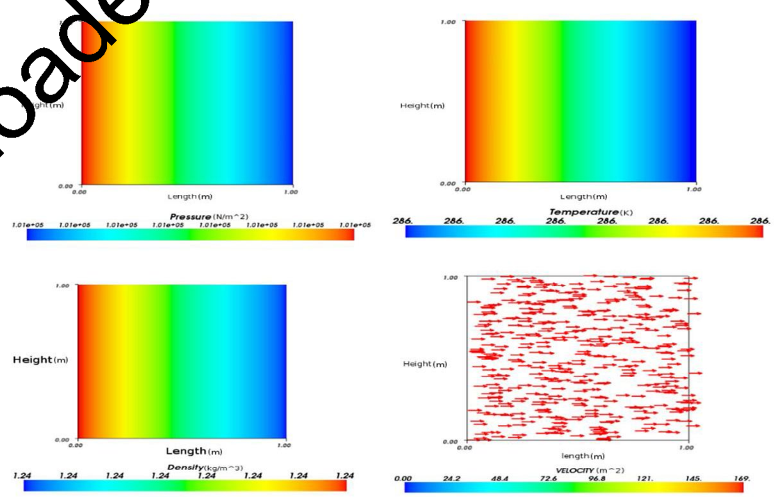
$$v_{Ghost} = v - 2(\vec{V}.\hat{n}).n_y$$

Scheme

We tried to use the 1d scheme but couldn't attain the desired solution so we tried the modified form of lax-fried rich scheme,"(16)"

$$Q_{ij}^{n+1} = \frac{Q^n_{i+1\,j} + Q^n_{i-1\,j} + Q^n_{i\,j+1} + Q^n_{i\,j-1}}{4} - \lambda(\vec{F}.\hat{n}\,ds)$$

Solution comparing the analytical with the error in a 2d solver



Solver results for subsonic pressure, temperature, density, velocity.

Chapter 5

Problems taken into case study are as follows

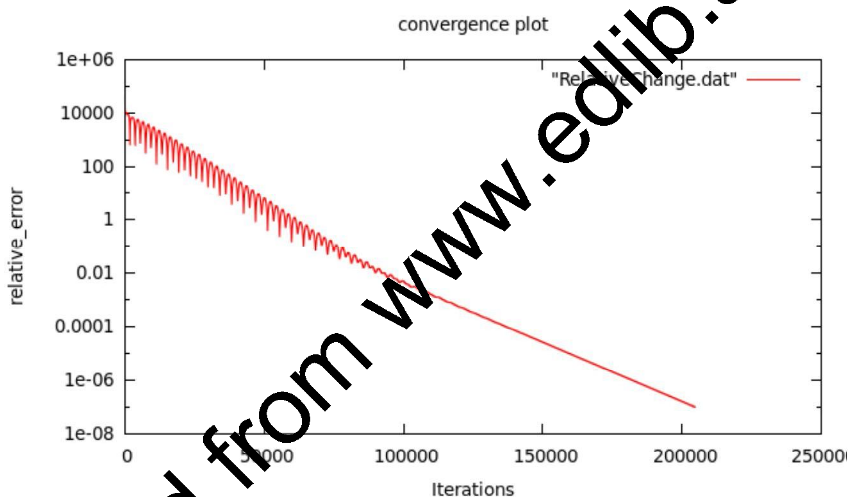
Flow over a Ramp

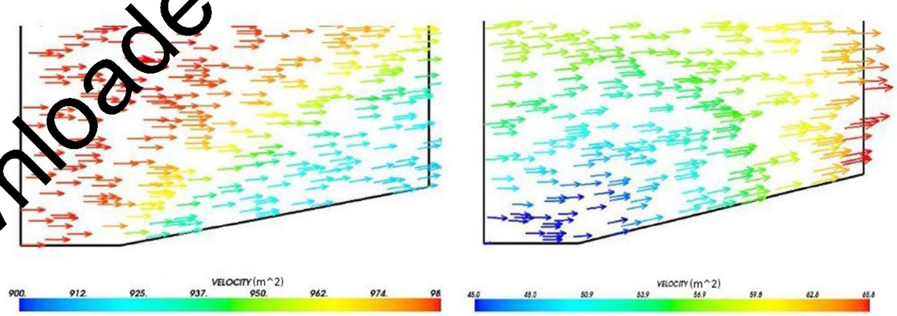
The solver is made to compute the flow over the ramp at mach 2.9 and mach 0.5 hexahedral structural mesh with different grid sizes is used and flow is visualized.

Boundary conditions:
Subsonic flow:
mach number 0.5

Supersonic flow:
mach number 2.9

Convergence: Target is 10^-7 and hexahedral mesh of 200*200 is used.

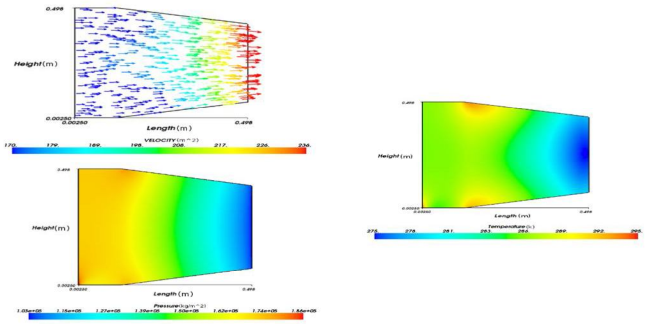




Flow over a ramp where its density, velocity vector for supersonic, velocity vector for subsonic is shown above.

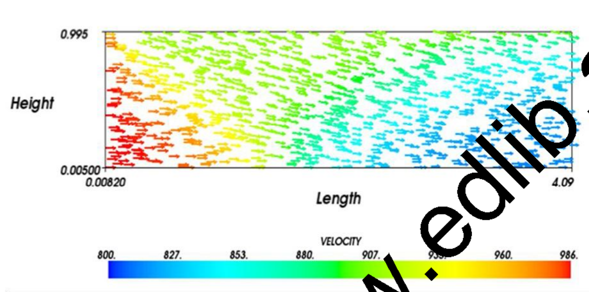
Flow in A Convergent Duct

The 2d geometry is shown in fig.the thickness in the z direction was taken as the value equal to the thickness in x direction.



Solver results of velocity, pressure and temperature.

Flow visualization of shock impinging on a flat plate



Solver results of shock impinging over a flat surface.

Chapter 6

Conclusion

Thus our solver has been proven to be working with supersonic and subsonic flows cases and the flow properties such as the values of change in velocities, temperature, pressure, and the shock and expansion effects have been clearly visualized and the value of fluid properties error has been considerably reduced. The reflection effect of shock waves is also clearly visualized, but the solver doesn't work for interaction of shock waves or reflection of expansion fans, changing the scheme may counter act these factors.

Reference Authors

1. Computational fluid dynamics: principles and applications, J. Blazek Elsevier, 2001
2. Computational fluid dynamics T. J Chung
3. Cambridge university press, 2002, first edition
4. Computational gasdynamics, lanley, c.b, volume 1
5. Cambridge university press, 1998, first edition
6. Fundamentals of aerodynamics, J. D Anderson, volume 1
7. Tata McGraw Hill, 2010, 6th edition
8. Gasdynamics, radhakrishnan. m, volume 1
9. Prentice of hall of india, 2006, 5th edition
10. Gas turbines, Ganesan .V, Lakshmi publications, third edition
11. Aerodynamic characteristics of transonic and supersonic flow over rectangular cavities, dangguo yang, jianqiang li zhaolin fan, danyao
12. Experimental study into the flow of physics of 3 d shock control bumps, P. J. K Bruce, H. Babinsky
13. Numerical simulation for reflection of oblique shock waves, Balamurugan M.