Investigations on the flexibility of a Group Sequence and its effect on the Best-Case Schedule

Zakaria YAHOUNI, Nasser MEBARKI and Zaki SARI
LUNAM, University of Nantes, « IRCCyN », Institut de Recherche en Communications et Cybernétique de Nantes, 44300 – France
University of Tlemcen, « MELT », Manufacturing Engineering Laboratory of Tlemcen, 13000, Algeria

***Abstract*-The group sequence is a well-known proactive-reactive scheduling method that brings sequential flexibility to an initial schedule in order to absorb uncertainties. Hence, group sequence favors the cooperation between human and machine. This method guarantees a minimal quality corresponding to the worst-case. This indicator associated with the best-case schedule provides the decision maker two bounds helping him to choose which operation will be executed first. In this paper, we consider the effect of the group sequence flexibility on the best-case schedule. The experiments made on very well-known instances of the job shop problem, using the makespan objective, brought two new results. The first one is that the best-case schedule of a group sequence is not guaranteed to be an optimal solution for the job shop problem, and the second one is that the more flexibility in a group sequence the better is the quality of the best-case.**
***Keywords*- Group Sequence, Flexibility, Best-case, Scheduling, Job Shop, Makespan.**

I. Introduction

In industrial scheduling problems, uncertainties, (e.g., breakdowns of machines, late material, new orders to proceed immediately…), are frequent. During the execution of the initial schedule, it is necessary to repair the schedule in real time while preserving the solution's quality. For this, scheduling methods which provide flexible solutions taking into account the uncertainties of the workshop are very interesting. One of the most studied scheduling methods bringing flexibility is the group sequencing method, Ref. [1]. This method aims at describing a set of feasible schedules in order to delay decisions to take into account uncertainties. Group sequencing is used according to two stages: a predictive phase and a reactive phase.

The predictive phase is done offline. It aims at introducing flexibility in the sequence of operations by creating groups of permutable operations which enables to describe a set of schedules without enumerating them. Then, the reactive phase is done online on the floor. It needs the intervention of a human, named the operator, who chooses during the execution of the group schedule the operation to be executed in each group of permutable operations that fits best the real state of the system, this method has been successfully addressed in the literature Ref. [2, 3, 4, 5, 6, 7, 9, 10, 11].

A group sequence guarantees a quality of the schedule that corresponds to the worst-case schedule; this value can be computed in a polynomial time for regular objectives Ref. [2, 3] and can be very helpful to evaluate a decision during the execution of the schedule. However, the best-case quality of a group sequence can also be interesting by providing to the decision maker two bounds, i.e., the minimal and the maximal quality of the schedule (Z_{worst} and Z_{best} resp.), Ref. [4]. Reference [5] developed a branch and bound algorithm for the best-case schedule in a group sequence based on an adapted lower bounds for the problem Ref. [6], this algorithm was experimented on benchmark instances. The optimal solutions of these instances were used as initial solutions for the construction of the group sequence. For this, the value of the best-case schedule of the group sequence was guaranteed to be an optimal solution regardless the flexibility of the group sequence. But the base-case quality of a group sequence is not always an optimal solution for the scheduling problem. One of the most important parameter influencing the best-case quality is the flexibility's degree of the group sequence.

In this paper, we investigate the relation between the flexibility of the group sequence proposed and the best-case schedule using the *makespan* objective denoted by Cmax. We prove that the best-case schedule of a group sequence is not guaranteed to be an optimal solution of the job-shop problem considered if the initial schedule used to construct the group sequence is not optimal. We also prove that the more flexibility is introduced in the group sequence the better is the best-case schedule.

The paper is organized as follows: the second section presents the group sequence, in section three the flexibility's equation of a group sequence is described. Section four presents the results of our experiments. We conclude this paper in section five.

II. Group of permutable operations

Group sequence or group of permutable operations was introduced by LAAS-CNRS laboratory, Toulouse, France Ref. [1], this approach has been used in the ORDO software, it describes a set of valid schedules, without enumerating them. The objective of this method is to provide to the decision-maker a sequential flexibility during the execution of the schedule and to ensure a certain quality that is represented by the worst process case Ref. [7].

A group of permutable operations is composed of groups G_i , each group contains one or many operations that will be executed in the same resource $G_i \coloneqq \{O_i, O_2, \ldots, O_n\}$, n! denotes the number of permutations that can be represented from this group. A group sequence is said feasible if any permutation among all the operations of the same group gives a feasible schedule that satisfies all the constraints of the problem.

To illustrate this definition, let us study a job shop example where the problem is described in tab1 (pi, Mi and Γ- denote respectively the processing time, the machine assignment and the predecessor of the current operation)

Table 1 : *Job Shop* Problem

	Job$_1$			**Job**$_2$			**Job**$_3$		
O$_n$	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8	O_9
M$_i$	M_3	M_2	M_1	M_2	M_3	M_1	M_1	M_2	M_3
p$_i$	4	4	3	5	2	4	4	3	5
Γ$^-$	/	O_1	O_2	/	O_4	O_5	/	O_7	O_8

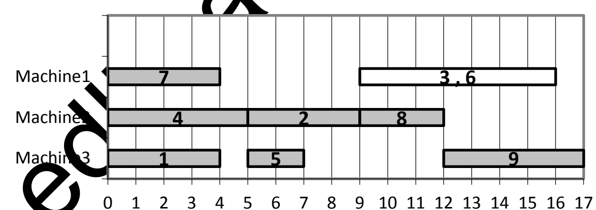

Figure 1 : Group Sequence solution

Figure 1 presents a feasible group sequence solving this problem. This group sequence is made of eight groups: one group of two operations and seven groups of one operation. Thus, the operator will have one decision to take in the reactive phase from two different semi-active schedules shown in Figure 2. In this group sequence, for the makespan the best-case quality is equal to the worst-case quality with Cmax = 17.

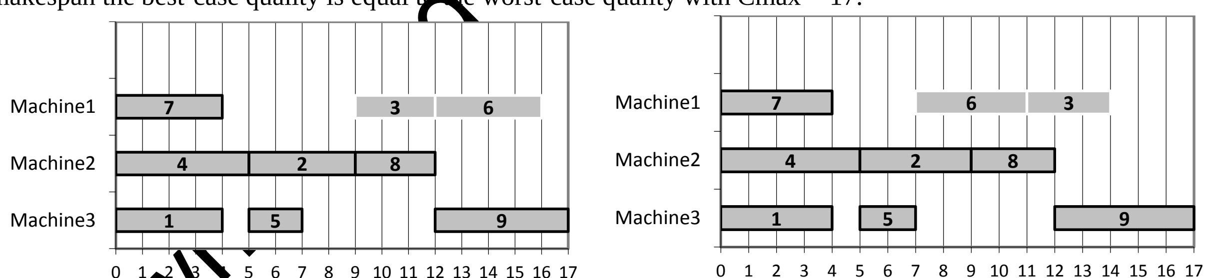

Figure 2 : Set of Semi-Active schedules

III. The Flexibility of a group sequence

The sequential flexibility of any group sequence is related to it groups number, this was summarized in Ref. [2], the more operations are grouped together the more choices will have the operator in the reactive phase; so to maximize the flexibility we minimize the number of groups. This measure is described in the next equation where #Gps, #Ops and #Ms denotes respectively the number of groups, the number of operations and the number of machines:

$$\Phi = \frac{\#Ops - \#Gps}{\#Ops - \#Ms} \tag{1}$$

Φ represents the grouping rate of a group sequence : the less flexible case is obtained when the number of operations in any group is equal to one, for this the numerator of the equation represented above will be equal to zero and the flexibility Φ = 0%. In the opposite way, when we have only one group by each machine (possible for flow shops) the flexibility will be at its maximum (Φ = 100%).

Figure 1 presents a group sequence composed of eight groups with Φ = 1/5 = 20%, the best-case that we obtained

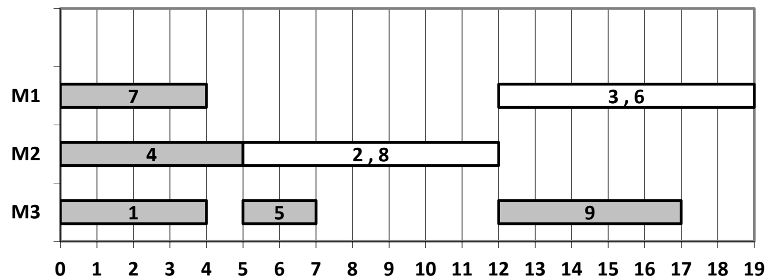

Figure 3 Group sequence with Φ = 50%

from this group sequence has a Cmax=17, this value can be improved by introducing more flexibility (regrouping more operations) in the group sequence, for example figure 3 represents a new feasible group sequence of the same *job shop* problem. This group sequence is represented by seven groups with Φ = 50% and describes four semi-active schedules (figure 4) with a best-case value equal to 15, this value is the optimal value for our *job shop* problem.

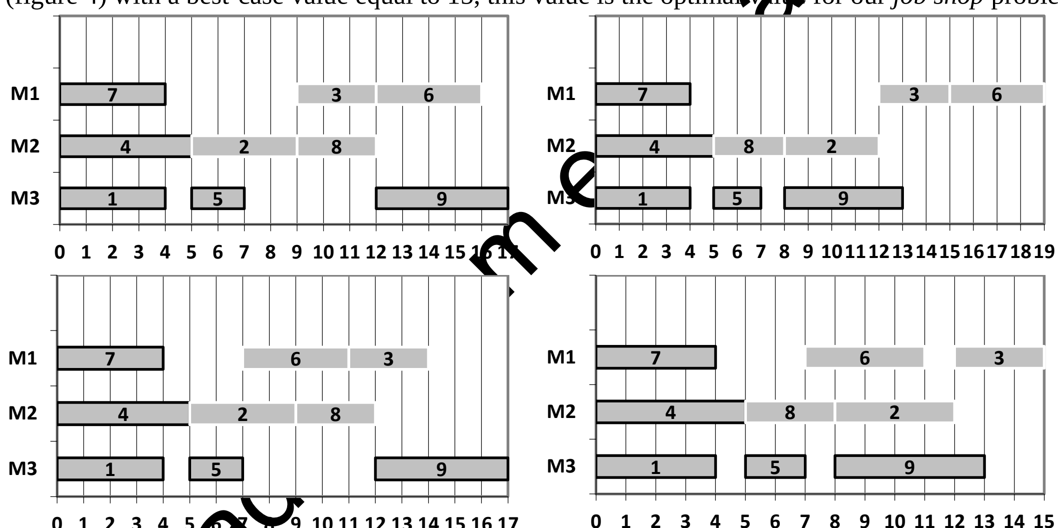

Figure 4 Set of semi-active schedules of Fig3

From this example, we proved that the best-case of a group sequence is not guaranteed to be an optimal solution of the initial problem; the more flexibility is introduced the same or better is the value of the best-case. But if the initial solution is optimal for the *job shop* problem, the best-case quality is optimal regardless the flexibility value Φ.

In the next section, we introduce different values of flexibility in a well-knows *job shop* instances to see their influence on the best-case schedule.

IV. Experiments and Discussion

5.1 Protocol

We took a well-known set of benchmark instances called la01 to la30 from Ref. [8]. These instances are widely used in the *job shop* literature. For each instance, we create an initial solution using the SPT dispatching rule, then from each initial solution we generate a group sequence with a given flexibility value using a greedy algorithm called EBJG that merges two successive groups according to different criteria until no group merging is possible. This algorithm begins with a one-operation-per-group sequence and is described in Ref. [2].

To compute the best-case quality a branch and bound algorithm is used with a depth_first search strategy Ref. [9]. This B&B algorithm allows avoiding the enumeration of all semi-active schedules of a group sequence

The experiments are made on an Intel(R) Core(TM) i7-3770 CPU 3.4GHz. The results according to the flexibility values are shown in the next table.

5.2 Results

Table 2 Experimentation results

Instances	Optimal Cmax	Initial Cmax	Φ ≈ 10%		Φ ≈ 20%		Φ ≈ 30%		Φ ≈ 50%	
			Best-case Cmax	CPU Time in seconds	Best-case Cmax	CPU Time in seconds	Best-case Cmax	CPU Time in seconds	Best-case Cmax	CPU Time in seconds
La01	650	748	748	*0,016*	748	*0,016*	721	*0,031*	699	*0,046*
La02	655	839	815	*0,016*	815	*0,031*	815	*0,047*	803	*0,047*
La03	597	809	809	*0*	809	*0,015*	809	*0,031*	763	*0*
La04	590	787	787	*0,016*	730	*0*	730	*0,016*	723	*9,36*
La05	593	593	593	*0,016*	593	*0,016*	593	*0*	593	*0*
La06	926	966	966	*0,031*	966	*0,016*	966	*0,016*	927	*0,016*
La07	890	1057	1057	*0,031*	1057	*0,016*	1002	*0,015*	1002	*0,015*
La08	863	1004	1004	*0,016*	1004	*0,015*	1004	*0,015*	1004	*0,031*
La09	951	1026	1026	*0*	1026	*0*	1026	*0,015*	1026	*0*
La10	958	978	973	*0,016*	973	*0,015*	973	*0,016*	973	*0,031*
La11	1222	1230	1230	*0,016*	1230	*0,046*	1230	*0,047*	1230	*0,063*
La12	1039	1066	1066	*0,015*	1066	*0,015*	1066	*0,015*	1066	*0,047*
La13	1150	1161	1161	*0,016*	1161	*0,031*	1161	*0,015*	1161	*0,031*
La14	1292	1292	1292	*0,016*	1292	*0,016*	1292	*0,015*	1292	*0,031*
La15	1207	1404	1404	*0*	1404	*0,016*	1403	*1,372*	1403	*1,326*
La16	945	1268	1193	*0*	1190	*0,016*	1186	*0,015*	1125	*0,015*
La17	784	914	912	*0*	890	*0*	890	*0*	867	*0,015*
La18	848	1015	1011	*0*	1011	*0*	1011	*3,994*	907	*0,624*
La19	842	965	965	*0*	949	*0*	921	*6,723*	907	*150,1*
La20	902	1146	1146	*0*	1133	*0,016*	1133	*0,016*	1132	*52,463*
La21	1046	1388	1388	*0*	1292	*0,031*	1292	*0,016*	1222	*0,047*
La22	927	1390	1390	*0,016*	1390	*0,016*	1359	*0,031*	1276	*0,53*
La23	1032	1142	1142	*0,015*	1142	*0,015*	1142	*0,015*	1078	*0,312*
La24	935	1180	1179	*0,016*	1179	*0,031*	1179	*0,031*	/	/
La25	977	1510	1447	*0,015*	1374	*0,031*	1365	*0,016*	/	/
La26	1218	1426	1426	*0,031*	1426	*0,046*	1426	*0,078*	1394	*0,234*
La27	1252	1678	1678	*0,015*	1658	*0,047*	1658	*0,078*	1658	*0,125*
La28	1273	1564	1548	*0,032*	1548	*0,046*	1515	*0,063*	1463	*1,217*
La29	1202	1566	1566	*0,031*	1566	*0,14*	1566	*0,14*	1464	*3,588*
La30	1355	1608	1607	*0,015*	1570	*0,562*	1570	*0,608*	/	/

6 Discussion

The best-case values for almost all instances were found in less than one second except when Φ = 50%, La24, La25 and La30 were not solved after twenty four hours. The best-case values for La05 and La14 are the only optimal solutions found from all instances, this is due to the fact that for these instances the initial generated solution using the SPT rule is optimal. Even when we stretch Φ to its maximum for some instances, i.e., La02 and La03, (75.55 and 77.77 respectively) the best-case values are 766 and 646 respectively, none of this values is an optimal solution for the initial *job shop* problem.

Comparing the variations of the best-case values with the variation of the group sequence flexibilities, we note that the best-case quality is improved when there is more flexibility. Figure 5 illustrates this variations for Φ = 10% and

Φ = 50%. Some instances like La05, La09, La10…etc keep the same values of the best-case for the different flexibility variations.

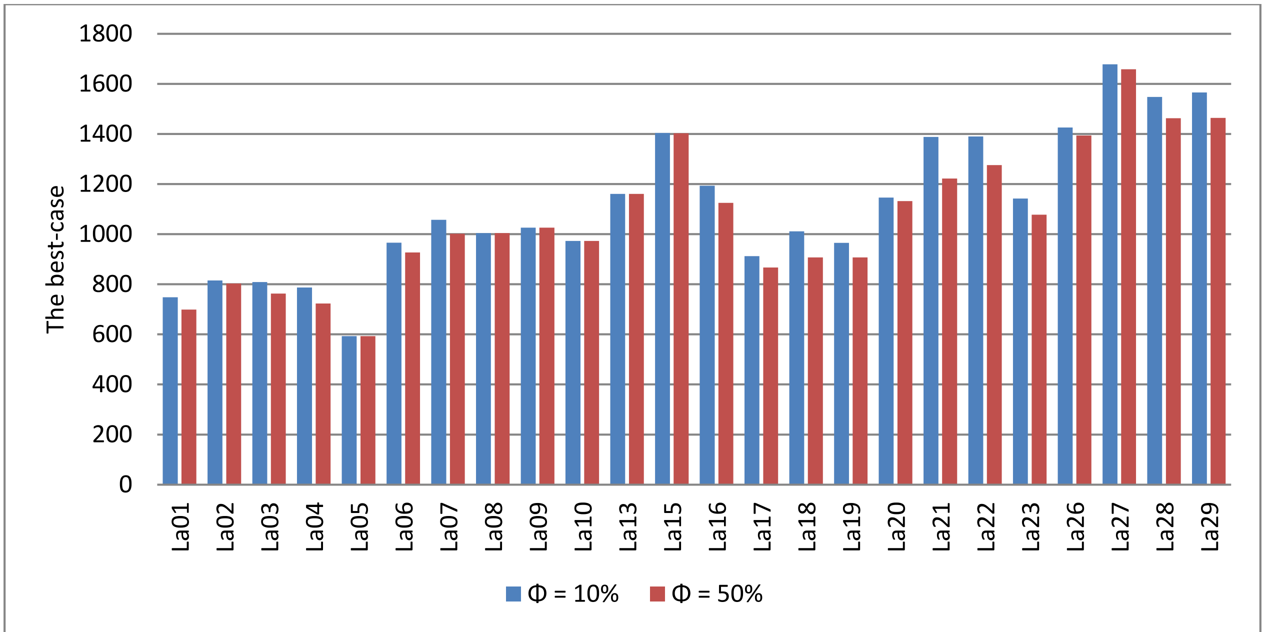

Figure 5 The variation of the best-case value regarding the group sequence flexibility

From these results, it comes that when the flexibility is higher the best-case schedule is equal or better. For the *makespan* objective this remark is represented in (2).

$$If\ (\Phi_2 > \Phi_1) \Rightarrow Z2_{best} \leq Z1_{best} \quad (2)$$

Conclusion

This paper investigates the effect of the flexibility of a group sequence on the best-case schedule for *job shop* problems, with *makespan* as objective. The best-case schedule was computed using an exact method, a branch and bound algorithm described in Ref. [5]. Experiments were conducted on thirty instances used in the *job shop* literature as a benchmark. The results show two main ideas: the first one is that the best-case of a group sequence depends on the initial solution and is not guaranteed to be optimal; the second one is that the best-case schedule may be improved when the flexibility of the group sequence is higher. However, the higher the flexibility is, the more difficult to find the best-case schedule. To investigate higher rates flexibility, we need to improve the branch and bound algorithm used to compute the best-case schedule. This can be explored in further research.

Acknowledgment

"This work has been funded with support of the European Commission. This communication reflects the view only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein."

REFERENCES

[1]. Erschler and Roubellat. 1989, An approach for real time scheduling for activities with time and resource constraints. s.l. : In Slowinski, R. and Weglarz, J., editors, Advances in project scheduling. Elsevier.

[2]. Carl ESSWEIN. 2003, Un apport de flexibilité séquentielle pour l'ordonnancement robuste. Université François Rabelais Tours (France).

[3]. Christian ARTIGUES, Jean-Charles BILLAUT and Carl ESSWEIN. 2005, Maximization of solution flexibility for robust shop scheduling. European Journal of Operational Research, Vol. 165, pp. 314-328.

[4]. Nasser MEBARKI, Olivier CARDIN and Guérin CLEMENT. 2013 , Evaluation of a New Human-Machine Decision Support System for Group. Las Vegas, : Analysis, Design, and Evaluation of Human-Machine Systems, August 13-15, Vol. 12, pp. 211-217.

[5]. Guillaume PINOT and Nasser MEBARKI. 2009, An Exact Method for the Best-case in Group Sequence: Application to a General Shop Problem. Moscow, Russia : 13th IFAC Symposium on Information Control Problems in Manufacturing.

[6]. Guillaume PINOT and Nasser MEBARKI. 2008, Best-case lower bounds in a group sequence for the job shop problem. Seoul, Korea : 17th IFAC World Congress.

[7]. Mohammed Ali ALLOULOU and Christian ARTIGUES. 2007, Worst-Case Evaluation of Flexible Solutions in Disjunctive Scheduling Problems. Malaysia : Gervasi, Osvaldo; Gavrilova, Marina, August 26-29, 2007, Computational Science and Its Applications - ICCSA 2007 International Conference, Kuala Lumpur, , August 26-29. Proceedings, Part III, pp. 1027-1036.

[8]. Lawrence, S. 1984, Resource constrained project scheduling: an experimental investigation of heuristic scheduling techniques (Supplement). Carnegie-Mellon University, Pittsburgh, Pennsylvania : s.n..

[9]. Guillaume PINOT. 2008, Coopération homme-machine pour l'ordonnancement sous incertitudes. Université de Nantes (France).

[10]. Rasaratnam LOGENDRAN. and Sara CARSON and Erik HANSON, Group scheduling in flexible flow shops. International Journal of Production Economics, Vol. 96, pp. 143-155. 2005.

[11]. Olivier CARDIN., Nasser MEBARKI., and Guillaume PINOT. A study of the robustness of the group scheduling method using an emulation of a complex fms. International Journal of Production Economics, 146:199-207. (2013).