Control of chaotic satellites systems based on the predictive control and passive control

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Abstract - In this paper, the problem of attitude stabilization was discussed by using the predictive control and the passive control theory. These controls are applied to the attitude equation of satellite which includes some types of nonlinear behavior such as periodic trajectory, chaotic dynamics. Moreover, the conditions under which the chaotic systems can be asymptotically controlled to the origin are presented. Simulation results are provided to show the effectiveness of the proposed control methods.

Keywords: Chaotic satellite attitude, Predictive control, passive control.

I. INTRODUCTION

In recent years, chaos theory has drawn great attention because of its theoretical importance and application in attitude dynamics of spacecraft. Chaotic systems are characterized by being extremely sensitive to initial conditions, deterministically random, and hence ultimately unpredictable.

In order to control the chaotic evolution of these systems, many nonlinear control methods have been developed. Feedback linearization technique to spacecraft attitude control has been discussed in [1-2]. Impulsive control has been used in [3-6]. Generalized Predictive Control approach has been used in [7]. A sliding-mode control has been used in [8]. Lyapunov-based control technique has been utilized in [9] and LMI-based nonlinear control in [10]. Nonlinear H∞ control in [11].

In this paper, we investigate the stability of chaotic satellite attitude control (angular velocities and attitude angles) using the feedback predictive control and the passive control methods. Some new sufficient conditions are derived for stabilizing the chaotic system. Finally, a comparison between the results obtained by the two proposed control is provided, it is based on the same initial conditions and the same moment of inertia.

II. STABILISATION OF SATELLITE

In this paper, the passive and the feedback predictive control are used to stabilize the satellite attitude given by the equations (1) and (2).

Satellite attitude kinematic equation is as follows:

\[
\begin{align*}
\dot{\phi} &= \omega_x + \sin \phi \tan \omega_y + \cos \phi \tan \omega_z \\
\dot{\theta} &= \cos \omega_y - \sin \phi \\
\dot{\psi} &= \sin \phi \sec \theta \omega_z - \cos \phi \omega_y
\end{align*}
\]  

The angular velocities are determined by a system of first order differential equations (Euler equations):

\[
\begin{align*}
I_x \omega_x &= (I_z - I_y) \omega_y \omega_z + C_x \\
I_y \omega_y &= (I_x - I_z) \omega_x \omega_z + C_y \\
I_z \omega_z &= (I_x - I_y) \omega_x \omega_y + C_z
\end{align*}
\]  

where \( I_x, I_y, I_z \) are the principal moments of inertia, \( \omega_x, \omega_y, \omega_z \) are angular velocities about the principal axes of the rigid body, and \( C_x, C_y, C_z \) are torques applied about these axes at time \( t \).

Earlier papers [14-16] have taken \( I_x = 3000 \text{ kg.m}^2, I_y = 2000 \text{ kg.m}^2 \) and \( I_z = 1000 \text{ kg.m}^2 \) with the perturbing torques defined by:

\[
\begin{bmatrix}
C_x \\
C_y \\
C_z
\end{bmatrix} = \begin{bmatrix}
-1200 & 0 & 1000 \sqrt{2}/2 \\
0 & 350 & 0 \\
-1000 \sqrt{2} & 0 & -400
\end{bmatrix} \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} + \begin{bmatrix}
1000 \cos \theta \sin \psi \\
1000 \cos \phi \sin \theta \\
1000 \cos \psi \sin \theta
\end{bmatrix}
\]  

These torques are chosen to be sufficiently large to induce very high chaotic motion and are comparable in magnitude with the available thrusters torques. The dynamics of the satellite will then exhibit chaotic motion [15].

The temporal evolution of the system defined by (1) and (2) is shown in Figures. 1 and 2.

A. Stabilisation using the passive control

Passivity is applied to nonlinear systems which are modeled by ordinary differential equations with input vector \( u(t) \) and output vector \( y(t) \) [13]:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)) \\
y(t) &= h(x(t))
\end{align*}
\]  

The system (4) is dissipative with the supply rate \( W(u(t), y(t)) \), if it is not able to generate power by itself, that is, the energy stored in the system is less than or equal to the supplied power.
$V(x(t)) \geq 0, \quad V(x(T)) - V(x(0)) \leq \int_0^T W(u(t), y(t)) \, dt$ \hspace{1cm} (5)

Furthermore, the storage function $V(x(t))$ must satisfy the requirements for a Lyapunov function.

\begin{align*}
&V(x(t)) \geq 0, \quad V(x(T)) - V(x(0)) \leq \int_0^T W(u(t), y(t)) \, dt \\
&\text{if there exists a positive semi definite Lyapunov function, such that:}
\end{align*}

$$
\int_0^T \frac{\partial V(x(t))}{\partial x(t)} f(x(t), u(t)) + \rho \dot{V}(x(t)) \, dt \\
+ \delta \dot{y}(t) y(t) \leq 0$$ \hspace{1cm} (6)

then the system (4) is passive. A passive system implies that any increase in storage energy is due solely to an external power supply.

Then the equilibrium point of the system: $\dot{x}(t) = f(t, x(t), 0)$ is asymptotically stable in either of the two cases:

(i) $\rho > 0$

(ii) $\epsilon + \delta > 0$ \hspace{1cm} (7)

The system (4) can be represented as the normal form [17]:

$$
\dot{z} = f(\bar{z}) + g(\bar{z}, y) y \\
\dot{y} = l(\bar{z}, y) y + k(\bar{z}, y) u
$$

The nonlinear system (8) may be rendered by a state feedback as the form:

$$
u = \alpha(\bar{z}) + \beta(\bar{z}) y
$$

We consider the system (1) and (2) as follows:

$$
\dot{\phi} = \omega + \sin \phi \tan \omega + \cos \phi \tan \omega \\
\dot{\theta} = \cos \phi \omega - \sin \phi \omega \\
\psi = \sin \phi \sec \omega + \cos \phi \sec \omega \\
\dot{w}_x = \left( I_x - I_z \right) \frac{1}{I_x} w_\omega w_z + \frac{1}{2I_x} w_z \frac{\cos \theta \sin \psi + T_x}{I_x} \\
\dot{w}_y = \left( I_x - I_z \right) \frac{1}{I_y} w_\omega w_z \frac{0.4}{I_y} w_z + \frac{\cos \phi \sin \theta}{I_x} T_x \\
\dot{w}_z = \left( I_x - I_z \right) \frac{1}{I_z} w_\omega w_z + \frac{0.4}{I_z} w_z + \frac{\cos \psi \sin \phi}{I_x} T_x \\
$$

\text{Suppose that state variable $w_y$ is the output of the system and suppose $w_z = z_1, w_y = y, w_x = z_2, \phi = z_3, \theta = z_4$ and $z_5$ then the system (10) can be expressed by passive form (8) where:}

$$
\begin{bmatrix}
\frac{1}{I_x}(-6z_1 - 0.4z_2 + \cos z_3 \sin z_1) \\
\frac{1}{I_y}z_3 + \cos z_3 \tan z_2 \\
-\sin z_2 \\
\frac{\cos z_3 \sec z_3}{I_z} \\
\sin z_2 \tan z_1 \\
\frac{\sin z_2 \sec z_1}{I_z}
\end{bmatrix}
$$

Then the system (10) can be expressed by passive form (8) where:

$$
\left[ \begin{array}{c}
\sigma_1 z_2 \\
\sigma_2 z_1 \\
\sin z_3 \tan z_4 \\
\cos z_3 \\
\sin z_2 \sec z_1
\end{array} \right] = \left[ \begin{array}{c}
f(z) \\
g(z, y) = \frac{1}{I_y}(z_1 + z_2 + z_3 + z_4 + z_5)
\end{array} \right]
$$

Our object is to design a smooth control (9) to make the closed-loop system passive.

Choose a storage function candidate:

$$
V(z, y) = W(z) + \frac{1}{2} y^2
$$

where $W(z)$ is Lyapunov function, with $W(0) = 0$ .

The Lyapunov function is:

$$
W(z) = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2 + z_4^2 + z_5^2)
$$

Derivative of the Lyapunov function is as follows:

$$
\frac{d}{dt} W(z) = z_1(-0.4z_1 + \frac{\sqrt{6}}{6} z_3 + \frac{1}{3} \cos z_3 \sin z_1) + z_2(\frac{\sqrt{6}}{3} z_4 - \frac{0.4}{I_z} z_2 \\
+ \cos z_3 \sin z_1) + z_3(\cos z_3 \tan z_2) \\
- z_4(\sin z_3) + z_5(\cos z_4 \sec z_3) \leq 0
$$
From equation (14), the zero dynamics of the chaotic system is asymptotically stable in the sense of Lyapunov. The derivative of $V(z, y)$ along the trajectory of system (10) is

$$\frac{d}{dt} V(z, y) = \frac{\partial}{\partial z} W(z)f(z) + \frac{\partial}{\partial z} W(z)g(z, y)y + l(z, y)y + k(z, y)yu$$

(15)

The system is minimum phase:

$$\frac{d}{dt} W(z)f(z) \leq 0$$

(16)

Equation (15) becomes:

$$\frac{d}{dt} V(z, y) \leq \frac{\partial}{\partial z} W(z)g(z, y)y + l(z, y) + k(z, y)u$$

(17)

If we select the feedback control (9) of the following form and consider (11):

$$u = k^{-1}(z, y)[-f'(z, y) - \gamma y + v]$$

$$= -(\sigma_3 + \sigma_4)z - \frac{0.35}{\Gamma_4} y - \gamma y - \frac{1}{\Gamma_4} \cos z_2 \sin z_2$$

$$- z_2 \cos z_2 \tan z_1 - z_2 \cos z_2 - z_2 \sin z_2 \sec z_1 + \nu$$

(18)

where $\gamma$ is a positive constant, and $v$ is an external signal which is connected with the reference input, the above inequality can be rewritten as

$$\frac{d}{dt} V(z, y) \leq -\gamma y^2 + vy$$

(19)

Then by integrating both sides of (19), we have

$$V(z, y) - V(z_o, y_o) \leq \int_0^t -\gamma y^2(\tau) d\tau + \int_0^t v(\tau) y(\tau) d\tau$$

(20)

If $V(z, y) \geq 0$ and $\rho = V(z_o, y_o)$ then

$$\int_0^t v(\tau) y(\tau) d\tau + \rho \geq V(z, y) + \int_0^t \gamma y^2(\tau) d\tau \geq \frac{\gamma}{\gamma} \rho \geq \rho$$

(21)

It satisfies the passive definition (6), the system (10) is rendered to be output strict passive (OSP) under the feedback control.

The system is simulated by the fourth-order Runge-Kutta integration method with the following initial conditions ($wx, wy, wz, \theta, \phi, \psi = (1, 1, 1, 1, 1)$) and use the controller as in (18).

The simulation results are shown in Figs. 3 and 4 for different values of $\gamma$. From the figures we have that the state variables quickly tend to the origin under passive control, the bigger $\gamma$ gives the best performance.

**B. Stabilisation using the predictive control**

Consider the nonlinear system described by [18-19]

$$\dot{x}(t) = f(x(t), u(t))$$

(22)

where $x \in R^n$ is the state vector and $u \in R^n$ is the feedback controller. We assume that $f$ is differentiable.

The control input $u(t)$ is determined by the difference between the predicted states and the current states:

$$u(t) = K(x(t) - x(t))$$

(23)

where $K$ is a gain vector, $x(t)$ is the predicted future state of uncontrolled chaotic systems from the current state $x(t)$.

Using a one-step-ahead-prediction, the predictive control (23) becomes

$$u(t) = K(\dot{x}(t) - x(t))$$

(24)
Near \( x_f \), we can use the linear approximation for the uncontrolled system by
\[
\dot{x}(t) - x_f = A(x(t) - x_f)
\]  
(25)
where \( A \in \mathbb{R}^{n \times n} \) is the Jacobian which is defined as follows:
\[
A = D_x f(x_f) = \left[ \frac{\partial x_i}{\partial x_j} \right]_{x = x_f}
\]  
(26)
The controlled system will be described by:
\[
\dot{\delta x}(t) = A \delta x(t)
\]  
(27)
Eq. (25) is rewritten in the form
\[
\delta \dot{x}(t) = x(t) - x_f
\]  
(28)
The controlled system is linearized around \( x_f \) by
\[
\delta \dot{x}(t) = A \delta x(t) + K \delta \delta x(t)
\]  
(29)
The feedback gain \( K \) is determined as follows [18]:
\[
A + K(A - I) < 1
\]  
(30)
And the vicinity of the fixed point is given by:
\[
r(t) = \left| x(t) - x(t-1) \right|
\]  
(31)
The controlled system will be described by [19]:
\[
\dot{\delta x}(t) = \begin{cases} 
 f(x(t)) + u(t) & \text{if } r(t) < \varepsilon \\
 f(x(t)) & \text{otherwise}
\end{cases}
\]  
(32)
Where \( \varepsilon \) is a positive small real number.

We consider the system as follows:
\[
\begin{align*}
\dot{w}_x &= \frac{(I_x - I_z)}{I_x} w_y w_z - \frac{1.2}{I_x} w_y + 3.6 \sqrt{6} w_x w_z - w_y \\
\dot{w}_y &= \frac{(I_z - I_x)}{I_y} w_x w_z - \frac{0.35}{I_y} w_z + K(I_x - I_z) w_y - w_z \\
\dot{w}_z &= \frac{(I_x - I_z)}{I_z} w_x w_y - \frac{0.35}{I_z} w_z - 0.4 \sqrt{6} w_x w_z 
\end{align*}
\]  
(34)
In order to control the system to the unstable equilibrium point \([0\ 0\ 0]^T\), we have to determine the correction which will be applied to the current state of the chaotic system. For this purpose, we determine the control input \( u(t) \) defined by Eq.(24).
\[
u(t) = K((\sigma_j w_y, w_z) + 0.35 w_x, -w_y)
\]  
(35)
The controlled system is given by:
\[
\begin{align*}
\dot{w}_x &= \frac{(I_x - I_z)}{I_x} w_y w_z - \frac{1.2}{I_x} w_y + 3.6 \sqrt{6} w_x w_z - w_y \\
\dot{w}_y &= \frac{(I_z - I_x)}{I_y} w_x w_z - \frac{0.35}{I_y} w_z + K(I_x - I_z) w_y - w_z \\
\dot{w}_z &= \frac{(I_x - I_z)}{I_z} w_x w_y - \frac{0.35}{I_z} w_z - 0.4 \sqrt{6} w_x w_z 
\end{align*}
\]  
(36)
Its linearized system around the fixed point is given by:
\[
\begin{align*}
\delta \dot{w}_x(t) &= \frac{\delta w_x(t)}{\delta w_y(t)} \\
\delta \dot{w}_y(t) &= \frac{\delta w_y(t)}{\delta w_z(t)} \\
\delta \dot{w}_z(t) &= \frac{\delta w_z(t)}{\delta w_y(t)} 
\end{align*}
\]  
(37)
Then, \( K \) must satisfy the inequality
\[
\frac{0.35}{I_y} + K(I_x - I_z) < 1
\]  
(38)
This implies that:
\[
K < 1.424
\]  
(39)
And the vicinity of the fixed point is given by:
\[
r(t) = \left| w_y(t) - w_x(t-1) \right|
\]  
(40)
Thus, the controlled system is described by:
\[
\begin{align*}
\dot{w}_y(t) &= K((\sigma_j w_y, w_z) + 0.35 w_x, -w_y) \\
&\quad \text{if } \left| w_y(t) - w_x(t-1) \right| < \varepsilon \\
&= 0 \quad \text{otherwise}
\end{align*}
\]  
(41)
In the simulation process the initial states of system are \((w_x, w_y, w_z) = (1, 1, 1)\) and the gain vector is \( K = [0.05 0.0] \). The simulation results are shown in Figs. 5 and 6 for \( k=0.5 \) and \( K=3 \).

![Figure 5. Controlled angular velocities for k=0.5.](image-url)
From the simulation results, it shows that the time responses of the system under the proposed feedback predictive control converge quickly to zero if the values of $K$ are in the interval $[0, 1.424]$. And if the value of $K$ is out of the interval, the predictively controlled system cannot be stabilized.

The following figures show the variations of the angular velocities controlled by the feedback predictive control, passive control and the impulsive method based on the same initial conditions.

From figures (7), (8) and (9), the results for the passive stabilization were more satisfactory than those for feedback predictive control.

III. CONCLUSION

In this paper, a feedback predictive control and a passive control have been proposed to stabilize the attitude of satellite. Necessary and sufficient conditions for stabilization are given. An illustrative example is finally included to visualize the effectiveness of each control. At the end a comparison between the simulation results is presented.

REFERENCES


