Abstract—This paper describes the simulation of a hybrid secure communication circuit with VHDL-AMS. The hybrid chaos synchronization strategy is developed from the point of view of the observer design, where the drive is a combination of a continuous-time hyper-chaotic (5D) system and a discrete time chaotic system (Henon), the response is a composed of a continuous unknown input observer and a discrete full-order state observer. Simulation results show the effectiveness of the proposed approach.

Keywords: hyperchaotic system, Chaos synchronization, observer, VHDL-AMS.

I. INTRODUCTION

Synchronization of hybrid chaotic systems and its application to secure communication have received considerable attention over the last decade. Since the pioneering work performed by Pecora and Carroll [1], different chaos communication methods have been developed in order to hide the contents of a message using hybrid chaotic signals.

An attractive way to simulate such complex systems in a reasonable amount of time is to use behavioral models to simplify physics and explore interaction between different domains. A modeling environment naturally suited for behavioral modeling of mixed technology problems is VHDL-AMS [3-5]. This high-level hardware description language is an IEEE standard and extension of a digital language VHDL [6]. VHDL-AMS is widely used in electronic design flow for designing various mixed-signal (analog and digital) circuits and systems including such recent applications as RFID systems [7].

This paper describes the simulation of a hybrid secure communication circuit with VHDL-AMS. In the transmission scheme, we propose a transmitter system combined of a continuous-time hyper-chaotic (5D) system and a discrete-time chaotic system called modified Henon. To make its structure more complex, the states of the continuous-time system are introduced in the dynamic of the discrete-time system. The receiver is composed from a continuous unknown input observer and a discrete full-order state observer. Simulation results are finally presented to visualize the satisfactory synchronization performance.

II. THE HYBRID CHAOS SYNCHRONIZATION SYSTEM

The continuous system is described by the following equations:

\[
\dot{z} = A_1 z + f(z, s_1, y_1) + B_1 s_1
\]

\[
y_1 = C_1 z + D_1 s_1
\]

\[
\dot{y}_2 = z + f(z, s_1, y_1) + B_1 s_1
\]

\[
y_2 = C_1 z + D_1 s_1
\]

\[
z \in \mathbb{R}^n, y_1 \in \mathbb{R}^m, s_1 \in \mathbb{R}^m \text{ denote the state, output and information signal respectively. } A_1, B_1, C_1 \text{ and } D_1 \text{ are real known matrices. } f(z, s_1, y_1) \text{ is the nonlinear item of the system.}
\]

For simplicity of the presentation we introduce the following notations:

\[
E = [I_n \ 0], M = [A_1 \ B_1], H = [C_1 \ D_1] \text{ and } \varphi = \begin{pmatrix} z \\ s_1 \end{pmatrix}
\]

The discrete-time system is described by the following equations [9]:

\[
x(n+1) = A_2 x(n) + B_2 f_2(x(n)) + C_2 + B s_2(n)
\]

\[
y_2(n) = d^T x(n) + K f(x(n)) + s_2(n) = \dot{z}(n) + s_2(n)
\]

\[
x \in \mathbb{R}^n, y_2 \in \mathbb{R}^m \text{ and } s_2 \in \mathbb{R} \text{ denote the state, output and information signal respectively. } A_2, B_2 \text{ and } C_2 \text{ are real known matrices. } f_2(x(n)) \text{ is the nonlinear item of the system.}
\]

We introduce in the dynamics of discrete-time system, the states \(z_1, z_2, z_3, z_4\) and \(z_5\) of the continuous-time system, sampled with a rate \(T_1\), to make the structure of the discrete system more complex [2].

The signal \(y_1\) comes from the continuous-time system will be first sampled with a period \(T_2\), but only blocked during \(T_1\). The signal \(y_2\) comes from the discrete-time system, is sent during 9\(T_1\). We obtain a transmission cycle composed of 10 periods \(T_1\).
The Discrete-time receiver system is described by:

\[ \hat{x} = N \hat{y} + Ly_1 + g(\hat{y}, y_1) \]

where \( \hat{x} \) denotes the state estimation vector of \( x \). Matrices \( B_0, d \) and \( K \) should be determined such that \( \hat{s}_2 \) converges to \( s_2 \).

Defining the synchronization error:

\[ e_2 = \hat{x} - x \]  \hspace{1cm} (7)

And if the following condition are verified,
- \( B_0 = b/K \)
- \( K \neq 0 \)
- \( d^T \) satisfies: \( \lambda_i(A - bd^T/K) < 1 \) for \( i = 1, \ldots, n \)

We can recover the message \( s_2 \).

The received signal is first demultiplexed on two signals \( y_1 \) and \( y_2 \). The signal \( y_1 \) is the first memorized during a period \( T_2 = 10T_1 \). Then, the signals \( y_1, y_2 \) are introduced, respectively, in continuous and discrete observers.

### III. VHDL-AMS SIMULATION

In this paper, all VHDL-AMS simulations have been simulated with the simulator HAMSTER. The continuous time chaotic system has initial conditions:

\( z_i(0), z_{i+1}(0), z_{i+2}(0), z_{i+3}(0), z_{i+4}(0) = (1, 3, 1, 0.5, 2) \)

and the discrete time has:

\( x_i(0), x_{i+1}(0), x_{i+2}(0) = (0, 1, 0.1, 0.1, 0.1) \)

All of these numerical experiments were performed using the fourth-order Runge-Kutta integration algorithm with an integration step of 0.00001s.

The equations of the continuous time hyper-chaotic 5D system are as follows [10]:

\[ \dot{z} = \begin{bmatrix} -a_1 & a_1 & 0 & 0 & 0 \\ a_2 & a_2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -a_3 & 0 \\ -a_5 & 0 & 0 & a_4 & -a_4 \end{bmatrix} z + \begin{bmatrix} 2z_2z_3z_4z_5 \\ -z_2z_3z_4z_5 \\ -z_2z_3z_4z_5 \\ z_2z_3z_4z_5 \\ z_2z_3z_4z_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 30 \\ 0 \\ 0 \\ 0 \end{bmatrix} s_1 \]

\[ y_1 = [0 1 0 0 0] z + s_1 \]

Where \( a_1 = 37, a_2 = 14.5, a_3 = 10.5, a_4 = 15, a_5 = 9.5 \).

The first transmitted information signal is: \( s_1(t) = 0.5 \sin(60\pi t) \). The discrete time chaotic system used is the modified Henon given by:

\[
\begin{align*}
x(n+1) &= \begin{bmatrix} 0 & 0 & -b \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(n) + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} x_2^2(n) \\
&+ \begin{bmatrix} a \\ 0 \\ \alpha_1 x_1(n) + \alpha_2 z_2(n) + \alpha_3 z_3(n) + \alpha_4 x_4(n) + \alpha_5 z_5(n) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ s_2(n) \end{bmatrix}
\end{align*}
\]
\[ y_2(n) = [0 \ 0 \ 0.1 \ 0] x(n) + x_2^2(n) + s_2(n) \\
= \xi(n) + s_2(n) \]

With \( a = 1.76 \), \( b = 0.1 \).

The coefficients of continuous states are chosen as: \( \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.0001 \).

The second transmitted information signal is: \( s_2(t) = 0.5 \sin(60\pi t) \).

Matrices P, Q, N and L of the continuous unknown input observer are:

\[
P = \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 \\
    0 & -1 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
Q = \begin{bmatrix}
    0.12 \xi_1 \\
    \xi_2 \\
    \xi_3 \\
    \xi_4 \\
    \xi_5 \\
\end{bmatrix},
\]

\[
L = \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    -30 \\
\end{bmatrix},
\]

\[
N = \begin{bmatrix}
    -37 & -35.84 & 0 & 0 & -72.84 \\
    -35.84 & -66.92 & 0 & 0 & -51.42 \\
    14.50 & -66.92 & 0 & 0 & 0 \\
    0 & 0 & -100.0 & 0 & 0 \\
    0 & 7.66 & 0 & -10.50 & 0 \\
\end{bmatrix}
\]

Initials conditions are given as:
\[
(\hat{z}_1(0), \hat{z}_2(0), \hat{z}_3(0), \hat{z}_4(0), \hat{z}_5(0)) = (1, 2, 3, 4, 5).
\]

Initial conditions of the discrete full-order state observer are given as:
\[
(x_1(0), x_2(0), x_3(0)) = (-3, 0.2, 0.1).
\]
Figure (2) presents the VHDL-AMS code for this system. Figures (3)-(7) give the continuous states and their corresponding estimations, figures (9)-(11) give the discrete states and their estimations. We can note that all the states are perfectly estimated by the continuous and the discrete observer. Figures (8) and (12) show that the transmitted signals can be reconstituted successfully.

IV. CONCLUSION

This paper explores the circuit simulation of a new transmission scheme for chaos synchronization via hybrid dynamical system using VHDL-AMS. Several sufficient conditions for driving the synchronization error to zero and recovering the transmitted signals have been proposed. A typical illustrative example accompanied by their VHDL-AMS implementation and simulation results has shown satisfactory control performance.

REFERENCES