Radiation Pattern of Circular Disc Antenna Printed on Isotropic or Uniaxially Anisotropic Substrate

Siham BENKOUDA, Sami BEDRA, 
Electronics Department, University of Batna. 
Batna 05000, ALGERIA. 
bedra_sami@yahoo.fr

Imad BENACER, and Tarek FORTAKI 
Electronics Department, University of Batna. 
Batna 05000, ALGERIA. 
t_fortaki@yahoo.fr

Abstract— in this work, the influence of uniaxial anisotropy in the substrate on radiation pattern of circular microstrip antenna is investigated theoretically. The analysis approach is based on the spectral-domain method of moments in conjunction with the stationary phase method. A new concise expression is derived for computing the radiation electric field. The validity of the solution is tested by comparing the computed results with the experimental data. Finally, numerical results of the variations of resonant frequency, bandwidth and radiation patterns of the structure for high order mode, with respect to anisotropy ratio of the substrate, for several values of substrate thickness and patch radius are also presented.

Keywords—Circular-disk patch, galerkin approach, uniaxial anisotropic substrate, stationary phase method.

I. INTRODUCTION

Circular microstrip patch antenna has been found to be more advantageous with respect to other well-known patch antenna types due to its easy adaptability to circular polarization and wideband operations having small patch sizes. Thus it has been preferred in mobile and satellite communication applications [1].

Various designs and formulations for circular disk microstrip antenna structure having single patch or their arrays on isotropic layers have been presented in the literature [2].

With the increasing complexity of geometry and material property, designing these antennas requires more and more dedicated and sophisticated computer-aided-design (CAD) tools to predict the characteristics.

The method of moments (MoM) has been proven to be one of the most powerful CAD tools for solving this class of problems. Especially, the effects of uniaxial type anisotropy on the resonant characteristics of circular-disk microstrip antenna have been investigated [1, 3-4] due to availability of this type of substances such as Sapphire, Boron Nitride and Epsilam-10.

In a very recent study [2, 5-6], resonant frequencies of circular microstrip resonators on the uniaxial dielectric and also some other media types are determined depending on the full-wave analysis. However, the effects of uniaxial anisotropy on radiation the resonant frequency and the bandwidth characteristics of a circular patch microstrip antenna for different structural parameter cases have not been included in literature.

Figure 1. Geometry of circular-disk microstrip antenna.

The study of anisotropic substrates is of interest, however, the designers should, carefully check for the anisotropic
effects in the substrate material with which they will work, and evaluate the effects of anisotropy.

Anisotropy is defined as the substrate dielectric constant on the orientation of the applied electric field. Mathematically, the permittivity of a uniaxial anisotropic substrate can be represented by a tensor or dyadic of this form [7].

$$\varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_z & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

(1)

$\varepsilon_0$ is the free-space permittivity. $\varepsilon_x$ is the relative permittivity in the direction perpendicular to the optical axis. $\varepsilon_z$ is the relative permittivity in the direction of the optical axis.

Many substrate materials used for printed circuit antenna exhibit dielectric anisotropy, especially uniaxial anisotropy [7]. In the following, the substrate material is taken to be isotropic or uniaxially anisotropic with the optical axis normal to the patch.

$$E(\rho, \phi, z) = \begin{bmatrix} E_\rho(\rho, \phi, z) \\ E_\phi(\rho, \phi, z) \end{bmatrix} = \sum_{k_\rho=0}^{\infty} \sum_{m=0}^{\infty} e^{im\phi} \int_{0}^{\infty} k_\rho dk_\rho \tilde{H}_k(\rho k_\rho) \varepsilon_n(k_\rho, z)$$

(2)

$$H(\rho, \phi, z) = \begin{bmatrix} H_\rho(\rho, \phi, z) \\ -H_\phi(\rho, \phi, z) \end{bmatrix} = \sum_{k_\rho=0}^{\infty} \sum_{m=0}^{\infty} e^{im\phi} \int_{0}^{\infty} k_\rho dk_\rho \tilde{H}_k(\rho k_\rho) \varepsilon_n(k_\rho, z)$$

(3)

The relation which connects the current $K_n(k_\rho)$ on the conducting plate with the electric field in the interface corresponding:

$$e_n(k_\rho, z) = \tilde{E}_n(k_\rho)$$

(4)

Where $\tilde{E}_n(k_\rho)$ is the dyadic function of Green in the field of the vectorial transforms of Hankel [7-9]. The dyadic function of Green is factorized in a diagonal matrix, having always the same form and independent of the geometry of the radiant plate.

We determined the tensor of Green for the structures considered. The tangential field electric is null on the conducting plate, which leads to an integral equation. To solve the integral equation, one applied the procedure of Galerkin which consists in developing the unknown distribution of the current on the circular patch in series of basis functions [3, 7] exit of the model of the cavity. Boundary conditions require that the transverse components of the electric field vanish on the perfectly conducting disk and the current vanishes off the disk, to give the following set of vector dual integral equations:

$$E_\rho(\rho, z) = \int_{0}^{\infty} dk_\rho k_\rho H_n(k_\rho \rho) \varepsilon_n(k_\rho, z), \quad \rho < a$$

$$K_n(\rho) = \int_{0}^{\infty} dk_\rho k_\rho H_n(k_\rho \rho) \varepsilon_n(k_\rho, z), \quad \rho > a$$

(5)

(6)

The use of the method of the moments in the spectral field allowed the resolution of the system of dual integral equations. The method of the moments (procedure of Galerkin) allows the decomposition of the solution of an integral equation, according to a development of basis functions, which is expressed in the form of a series of functions as follows:

$$K_n(\rho) = \sum_{p=1}^{P} a_{np} \Psi_{np}(\rho) + \sum_{q=1}^{Q} b_{nq} \Phi_{nq}(\rho)$$

(7)

P and Q correspond to the number of basis functions of $\Psi_{np}(\rho)$ and $\Phi_{nq}(\rho)$, respectively, $a_{np}$ and $b_{nq}$ are the mode expansion coefficients to be sought. The corresponding VHT of the current is given by

$$K_n(\rho) = \sum_{p=1}^{P} a_{np} \Psi_{np}(\rho) + \sum_{q=1}^{Q} b_{nq} \Phi_{nq}(\rho)$$

(8)

Substitute the current expansion (8) into (5). Next, multiplying this one by $\rho^l \Psi_{nl}^+(\rho)$ ($k=1,2,..., P$) and by $\rho^l \Phi_{nl}^+(\rho)$ ($l=1,2,..., Q$), and while integrating from 0 to a, and using the Parseval’s theorem for the vectorial transforms of Hankel [7], we obtain a system of linear $P+Q$ algebraic equations for each mode n which can be written in the matrix form:

$$\mathbf{Z}_n C_n = 0$$

(9)
where:

\[
\mathbf{Z}_{\text{eq}} = \begin{bmatrix}
(\mathbf{Z}_{\text{eq}})^{PP}_{PP} & (\mathbf{Z}_{\text{eq}})^{PP}_{PQ} \\
(\mathbf{Z}_{\text{eq}})^{PP}_{QP} & (\mathbf{Z}_{\text{eq}})^{QQ}_{QQ}
\end{bmatrix},
\]

\[
\mathbf{C}_n = \begin{bmatrix}
(a_n)_{PQ} \\
(b_n)_{QQ}
\end{bmatrix}
\]

(10)

Each element of under matrix is given by:

\[
\mathbf{Z}_{\text{eq}}^{PW}(i, j) = \int_0^{+\infty} dk\rho \sqrt{V_m(k_p)} \sqrt{W_n(k_p)}
\]

(11)

where \( V \) and \( W \) represent either \( \Psi \) or \( \Phi \). For every value of the integer \( n \), the system of linear equations (9) has non-trivial solutions when

\[
\det[\mathbf{Z}_{\text{eq}}(\omega)]=0
\]

(12)

This equation (12) is called characteristic equation. For the research of the complex roots of this equation, the method of Müller is used. It requires three initial points which must be closer if possible to the solution sought to ensure a fast convergence.

Generally the real part \( f_i \) of the solution represents the frequency of resonance of the structure, whereas the imaginary part \( f_i \) indicates the losses of energy per radiation and the report \( (2f_i/f_r) \) gives the band-width and the quantities \( Q=(f_r/2f_i) \) indicated the factor of quality [7].

### III. NUMERICAL RESULTS AND DISCUSSION

In order to confirm the computational accuracy of the approach described in the previous section, Table 1 shows the results for the resonant frequencies and bandwidth of the different resonant modes of circular microstrip antenna on isotropic substrate have been compared with previously published results [3].

**TABLE I. COMPARISON OF RESONANT FREQUENCIES OF THE FIRST FOUR MODES OF A CIRCULAR MICROSTRIP ANTENNA ON A DIELECTRIC SUBSTRATE (\( \varepsilon_r=2.65, d=1.3875 \, \text{mm} \)).**

<table>
<thead>
<tr>
<th>Modes</th>
<th>Our Results of [3]</th>
<th>Our Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resonant frequency ( f_r ) (GHz)</td>
<td>Bandwidth BW (%)</td>
</tr>
<tr>
<td>TM(_0^1)</td>
<td>6.1703</td>
<td>5.23</td>
</tr>
<tr>
<td>TM(_0^2)</td>
<td>7.056</td>
<td>6.99</td>
</tr>
<tr>
<td>TM(_0^3)</td>
<td>10.381</td>
<td>6.13</td>
</tr>
<tr>
<td>TM(_0^4)</td>
<td>12.268</td>
<td>11.06</td>
</tr>
</tbody>
</table>

Fig. 2 depicts that the variation of dominant mode resonant frequency with the variation of patch radius \( a/h \). The antenna parameters for this studies are \( d=0.49mm, \varepsilon_r=2.43 \). Here we have compared our computed values with the measurements done by [3]. This plot indicates that the present model shows excellent agreements with the measurements for all values of patch radius.

Next the effect of uniaxial anisotropy on the resonant frequency, bandwidth and radiation pattern of the structure is analyzed.

The anisotropy ratio is defined as

\[
AR=\frac{\varepsilon_x}{\varepsilon_z}
\]

(13)

It is clear that anisotropy ratio take the value of ‘1’ for the isotropic case. In this study AR>1 type uniaxial anisotropy is called as positive anisotropy and AR<1 case is called as negative uniaxial anisotropy, it is clear that anisotropy ratio take the value of ‘1’ for the isotropic case [1].

Fig. 3(a) depicts the influence of the patch radius on the resonant frequency of a circular microstrip antenna for two anisotropic dielectric substrates: Boron nitride \( (\varepsilon_x=51.2, \varepsilon_z=3.4) \), Epsilam-10 \( (\varepsilon_x=13, \varepsilon_z=103) \). The substrate has thickness \( d=1.27mm \) and the radius patch is varied from 5mm to 50mm. As it can be seen, the resonant frequencies reduce considerably when the dielectric substrates change from Boron nitride to Epsilam-10. Also it observed that the resonant frequency increases with the patch radius. The effect of uniaxial anisotropic substrate on the bandwidth of circular microstrip antenna is also studied. The results in Fig. 3(b) present the variation of antenna bandwidth with two anisotropic dielectric substrates: Boron nitride and Epsilam-10.

In Fig. 4(a), the resonant frequency decreases with increasing anisotropy ratio, whereas in Fig. 5(a), the resonant frequency increases with increasing anisotropy ratio.
The reason for this apparent contradiction is that in Fig. 4(a) $\varepsilon_X$ is varied, so that the resonant frequency increases as $\varepsilon_X$ decreases. In Fig. 5(a), $\varepsilon_Z$ is varied, so that the resonant frequency increases as $\varepsilon_Z$ decreases.

Therefore, in both cases, as the permittivity being changed decreases, the resonant frequency increases.

In Figs. 4(b) and 5(b) the variation of antenna bandwidth in respect to anisotropy ratio is shown. It is observed that in order to increase the bandwidth to achieve wider bandwidths, anisotropy ratio ($AR = \varepsilon_X / \varepsilon_Z$) must be more than unity. Using an anisotropy ratio less than one leads to a high bandwidth suitable for resonator design.

In Fig. 7, the radiation pattern of the TM11 mode in the E plane ($y-z$ plane Fig. 7(a)) and in the H plane ($x-y$ plane Fig. 7(b)), as a function of anisotropic dielectric substrate, are shown.
Isotropic case ($\varepsilon_x=2.32, \varepsilon_z=2.32$)

Positive uniaxial case ($\varepsilon_x=2.32, \varepsilon_z=1.16$)

Negative uniaxial case ($\varepsilon_x=2.32, \varepsilon_z=4.64$)

Resonant frequency (GHz)

Radius of patch (mm)

(a)

Isotropic case ($\varepsilon_x=2.32, \varepsilon_z=2.32$)

Positive uniaxial case ($\varepsilon_x=2.32, \varepsilon_z=1.16$)

Negative uniaxial case ($\varepsilon_x=2.32, \varepsilon_z=4.64$)

Bandwidth (%)

Radius of patch (mm)

(b)

Figure 5. (a) Resonant frequency, (b) Bandwidth of circular microstrip antenna, versus patch radius, for different anisotropic dielectric substrates with substrate thickness $d=1.27\,\text{mm}$.

We also observe that the permittivity $\varepsilon_z$ has a stronger effect on the radiation than the permittivity $\varepsilon_x$.

Figure 6. Radiation patterns of the fundamental resonant mode of a circular disk microstrip antenna on uniaxial substrate for two different dielectric substrate materials, $h=1.27\,\text{mm}$, $a=10\,\text{mm}$. (a) E plane, (b) H plane.

(b)
The radiation pattern of an antenna becomes more directional as its $e_z$ increases. Another useful parameter describing the performance of an antenna is the gain.

IV. CONCLUSION

This work presents a full-wave analysis for the circular microstrip antenna printed on isotropic or uniaxially anisotropic media. The complex resonant frequency problem of structure is formulated in terms of an integral equation. Galerkin procedure is used in the resolution of the electric field integral equation, also the TM, TE waves are naturally separated in the Green’s function.

Numerical results concerning the effect of isotropic substrate on the characteristics of the antenna are presented. The effects of uniaxial substrate on the resonant frequency and radiation pattern of structures are considered in detail. It was found that the use of such substrates significantly affects the characterization of the microstrip antennas, and the permittivity $e_z$ along the optical axis has a stronger effect on the radiation of antenna. A comparative study between our results and those available in the literature shows a very good agreement. The accuracy of the method was checked by performing a set of results in terms of resonant frequencies and radiation patterns.

The analysis presented here can also be extended to study other parameters characterizing the circular patch antennas with various structures.

REFERENCES


