

# Effect of Transmitted Power per Pilot Symbol in LSE Channel Estimation for OFDM Systems

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**Abstract:** The Channel estimation is improving the BER performance in the OFDM systems and considers an important part in OFDM systems. It can be employed for the purpose of detecting received signal, improving the capacity of OFDM. In this paper, studied and analyzed the effects of channel estimation, by using Least-Squares Error (LSE) algorithm in OFDM system, where change in the transmitted power per pilot symbol in the FFT-OFDM system. A mathematical derivation of the algorithm LSE channel Estimation by control on the value of the transmitted power per pilot symbol is also presented.

**Keywords:** Channel estimation; Least-Squares Error (LSE); OFDM system.

## I. INTRODUCTION

In OFDM systems, channel estimation consists of estimating the channel gain at each of the frequency subcarriers. In the case of no Intercarrier Interference (ICI) and no Intersymbol Interference (ISI), the channel gain at each of the subcarriers is simply the channel frequency response evaluated at the corresponding subcarrier. However, in the case of ICI and/or ISI, the channel gain for each of the subcarriers is a distorted version of the channel frequency response at the corresponding subcarrier [1]. In the pilot-aided channel estimation methods, there are two classical pilot patterns, which are the block-type pattern and the comb-type pattern [2]:

### A. Block-Type Pilot Channel Estimation

A block-type channel estimation technique is performed by inserting pilot carriers in all sub-carriers of OFDM symbols. OFDM channel estimation pilots are transmitted periodically. It has been developed under the assumption of slow fading channels [3-5], the channel is stationary within a certain period of OFDM symbols. The channel estimation of Block-type pilot schemes are used over middle or fast fading channels, due to function of

the location, the channel estimation error may vary considerably of the data blocks with respect to the pilot block. The result may be a periodic variation of the decoding error rates for different OFDM blocks. This type of channel estimation can be based on least squares (LS) or minimum mean-square error (MMSE). The MMSE estimate has been shown to give (10-15) dB gain in SNR for the same mean square error of channel estimation over LS estimate [3-5].

### B. Comb-Type Pilot Channel Estimation

The comb-type refers to that the pilots are inserted at some specific subcarriers in each OFDM symbol, and it has been introduced whenever the channel changes fast, even in one OFDM block, that is, the channel varies over two adjacent OFDM symbols but remains stationary within one OFDM symbol. This is done in order to satisfy the need for channel equalization. This technique estimates the channel at pilot frequency and interpolates the channel for the block of data [3-5].

It is often performed by two steps. Firstly, it estimates the channel frequency response on all pilot subcarriers, by LS method, LMMSE method, and so on. Secondly, it obtains the channel estimates on all subcarriers by interpolation, including data subcarriers and pilot subcarriers in one OFDM symbol [6].

The LS estimator has a merit of low complexity but may suffer from noise, and the channel estimation for OFDM systems is usually carried out in frequency domain by the LS method using known pilot symbols. To enhance the noise immunity of the LS estimator, consider the estimation noise in time domain named discrete Fourier transform (DFT)-based channel estimation [7]. In any design of channel estimators for wireless OFDM systems, there are two main problems, the first problem is the arrangement of pilot information, where pilot means the reference signal used by both transmitters and receivers. The **second problem is the design of an estimator** with both low complexity and good channel tracking ability. The two problems are interconnected [8].

## II. LEAST-SQUARES ERROR (LSE)

In the OFDM transmission system the Information and pilot symbols are modulated on a set of subcarriers, and transmitted over a frequency-selective fading channel through a single transmitter antenna. After demodulation at the receiver end, where allow for multiple antennas, the channel per receive-antenna is estimated using pilots [9].

The Information and pilot symbols are modulated on a set of subcarriers in the transmitter side. The signal will be transmitted over a frequency-selective fading channel, and in the receiver side, the demodulation at the end.

In this section, using the same equations in [9] for the received signal corresponding to pilot symbols and the received samples corresponding to information symbols, but using the QAM modulation and the receive antenna is one as shown in

Figure 1, and the transmitted power per pilot symbol ( $\mathcal{E}_p$ ) is consider constant and equal 2. Derive exact Symbol Error Rate (SER) expressions for pilot assisted OFDM transmissions with QAM modulation over Rayleigh-fading channels. The SER analysis also quantifies the performance loss due to channel estimation error and the transmitted pilot power.

In the received signal corresponding to pilot symbols at the  $n^{th}$  subcarrier can be written [9]:

$$y[n] = \sqrt{\mathcal{E}_p} H(n)s(n) + w(n), \quad n \in I_p, \quad (1)$$

Where:

$I_p$  =set of subcarriers on which **pilot symbols** are transmitted,  
 $\mathcal{E}_p$  =transmitted power per **pilot symbol**,  $H(n)$  =channel frequency response at the  $n^{th}$  subcarrier,  
 $s(n), n \in I_p$  Is the pilot symbol,  $w(n)$  =complex AWGN with zero-mean and variance  $N_0/2$  per dimension;

Through the analysis and many results of the equation (1), and in several possibilities for get the better of performance SER when using the LSE channel Estimation. Where using of Equation (3) after the modification as in equation (2):

$$y[n] = \sqrt{2} H(n)s(n) + w(n), \quad n \in I_p \quad (2)$$

Where using the value of  $\mathcal{E}_p$  is fixed and equal 2.

Note the effect on the performance SER through the value of  $\mathcal{E}_p$  as in equation (1). Where the value of  $\mathcal{E}_p$ , it controls the equation (1)

This equation will become more convenient to show the performance SER using the LSE channel Estimation and it controlled with this equation through the value of  $\mathcal{E}_p$ .

While in the received samples corresponding to information symbols can be expressed as [9]:

$$y[n] = \sqrt{\mathcal{E}_s} H(n)s(n) + w_m(n), \quad n \in I_s \quad (3)$$

Where:

$I_s$  = set of subcarriers on which **information symbols** are transmitted,  $\mathcal{E}_s$  = transmitted power per **information symbol**,  
 Suppose that the total number of subcarriers is  $N$ , and the size of  $I_p$  is  $|I_p| = P$ .

For simplicity, assume that [9]:

Size of  $I_s$  is  $|I_s| = N - P$ , although it is possible that  $|I_s| < N - P$ , when null subcarriers are inserted for spectrum shaping.

Selecting information symbols from QAM constellations have also that  $|s(n)| = 1, \forall n \in I_s$ .

The frequency-selective channel is assumed to be Rayleigh-fading, with channel impulse response  $h := [h(0), \dots, h(L_t - 1)]^T$  corresponding to  $L_t$  = No. of taps; i.e.  $\forall l \in [0, L_t - 1]$  are uncorrelated complex Gaussian random variables with zero-mean [9].

Define the  $(L_t \times L_t)$  matrix  $[F]_{l,n} := \exp(j2\pi(l-1)(n-1)/N)$ , and let  $f_n$  = the  $n^{th}$  column of  $F$ .

$H(n) = f_n^* h$  is a complex Gaussian random variable with zero-mean. The average signal-to-noise ratio (SNR) per pilot symbol  $\mathcal{E}_p/N_0$ , and per information symbol  $\mathcal{E}_s/N_0$ . The AWGN variables  $w(n)$  are assumed to be uncorrelated  $\forall n$ , [9].

Suppose that the set of pilot subcarriers is given by  $I_p = \{n_i\}_i^p$  Letting [9].

$h := [H(n_1), \dots, H(n_p)]^T$  Contain the channel frequency response on pilot subcarriers, and defining  $F_p := [f_{n_1}, \dots, f_{n_p}]$

can relate the FFT pair via:  $h = F_p^H y$ .

Let the  $P \times 1$  vector  $y := [y(n_1), \dots, y(n_p)]^T$  consist of the received **pilot samples** per block, and define  $S_p := [s(n_1), \dots, s(n_p)]^T$ , and  $w := [w(n_1), \dots, w(n_p)]^T$ .

From (1), have [9].

$$y = \sqrt{\mathcal{E}_p} D(S_p) \tilde{h} + w = \sqrt{\mathcal{E}_p} D(S_p) F_p^H h + w \quad (4)$$

Using the pilot samples from only one block to estimate the channel on per block, because this is particularly suitable for packet data transmission, and the receiver may receive different blocks with unknown delays.

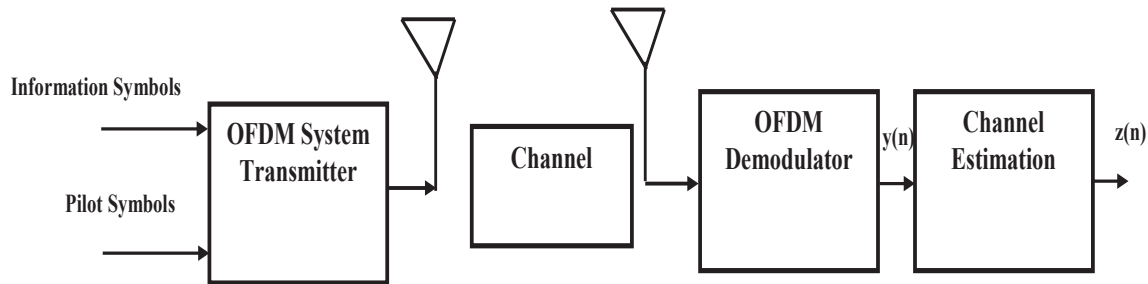


Figure 1: OFDM System.

### III. SIMULATION RESULTS

In this section, the simulation results are presented which show the performance of Least-Squares Error (LSE) channel estimation will be evaluated based on bit error rate in received data. The comb pilot aided channel estimation in OFDM modulation is considered and also the power of pilot subchannels is under the control in this simulation. The Symbol Error Rate (SER) is plot versus SNR, and the parameters of this simulation as shown in Table 1. The bit rate used in this simulation is 54 Mbps.

TABLE I. SIMULATION PARAMETER

Parameters	
Modulation QAM	8
Number of subcarriers ( $N_c$ )	256
pilot position interval	8
Number of Pilots (P)	256/8
Number of data subcarriers (S)	$S = N_c - P$
Transmitted power per information symbol	4 and 8
guard interval length (GI)	$GI = N_c/4$
Number of Taps (L)	16
Number of iteration in each evaluation	500

Figure 2, can show the performance of Least-Squares Error (LSE) Channel Estimation of OFDM Based on Comb Pilot, assume the Number of subcarriers  $N_c=256$ , and pilot position interval is 8, while the transmitted power per pilot symbol ( $\epsilon_p$ ) is equal 2.

Figure 3, can show the performance of LSE Channel Estimation of OFDM Based on Comb Pilot, assume the Number of subcarriers  $N_c=256$ , and pilot position interval is 8, while the transmitted power per pilot symbol ( $\epsilon_p$ ) is equal 2, 4 and 8. Can shown from this figure the performance of system when the transmitted power per pilot symbol ( $\epsilon_p$ ) equal 2, is better than when using the transmitted power per pilot symbol ( $\epsilon_p$ ) equal to 4 and 8.

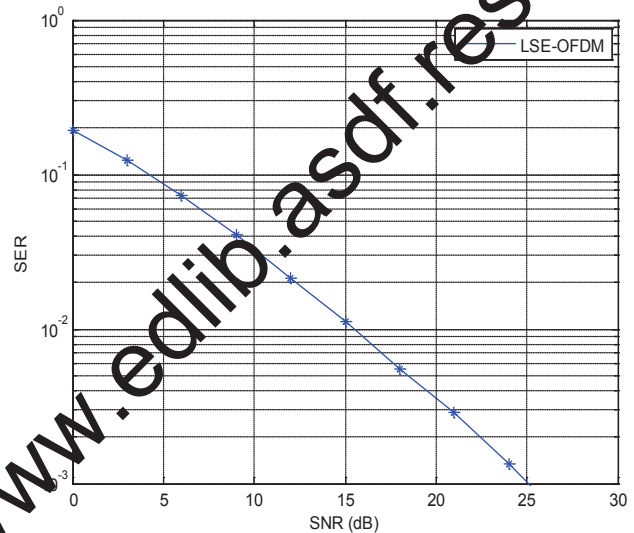


Figure 2: LSE channel estimation in FFT-OFDM,

$$N=256 \text{ \& } \epsilon_p = 2.$$

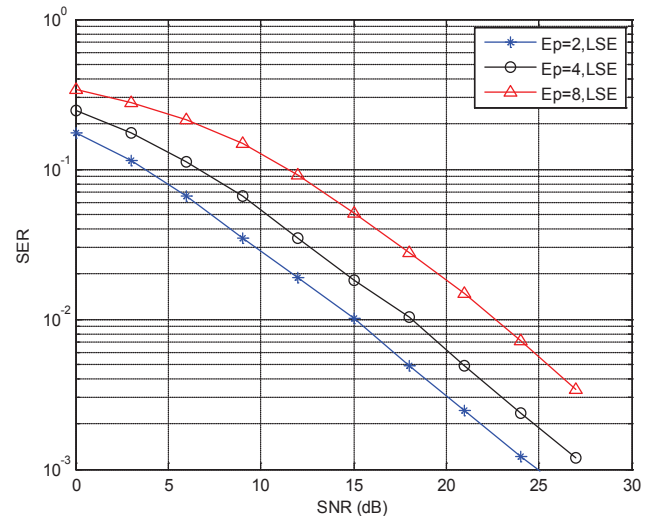


Figure 3: LSE channel estimation in FFT-OFDM,  $N=256$  &

$$\epsilon_p = 2, 4 \text{ and } 8.$$

#### IV. CONCLUSION

In this paper, the derived exact SER formulas, and quantified the performance loss due to channel estimation error and transmitted pilot power. Since the number and placement of pilot symbols, as well as the power allocation between the pilot and information symbols affect SER performance, optimized this parameters for LSE to minimize SER.

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